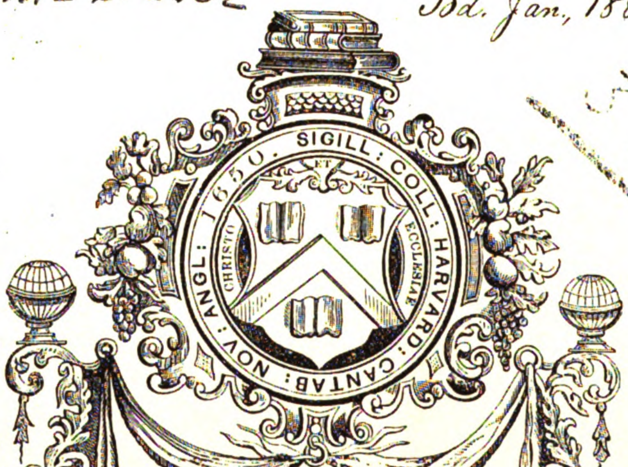

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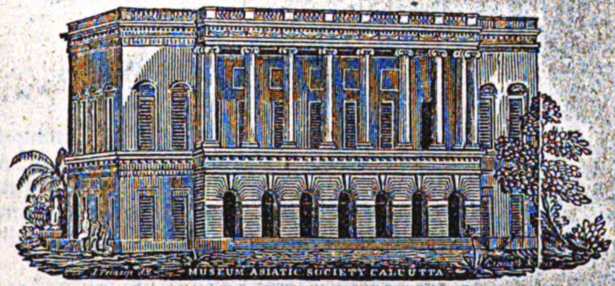

HINDU ASTRONOMY II.

THE SIDDHÁNTA S'IROMANI.

TRANSLATED FROM THE SANSKRIT BY THE LATE

LANCELOT WILKINSON, ESQ., C. S.

AND

REVISED BY PANDIT BÁPÚ DEVA S'ÁSTRÍ,

UNDER THE SUPERINTENDENCE OF THE VEN'BLE ARCHDEACON PRATT.

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TRANSLATION

OF THE

SIDDHÁNTA ŚIROMANI.

TRANSLATION

OF THE

SÚRYA SIDDHÁNTA

BY

PUNDIT BÁPÚ DEVA SÁSTRI,

AND OF THE

SIDDHÁNTA ŚIROMANI

By Bhāscara Ācārya.

BY THE LATE

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CONTENTS.

TRANSLATION OF THE GOLÁDHYÁYA OF THE SIDDHÁNTA-S'IROMAṆI.

CHAPTER I.

In praise of the advantages of the study of the Spheric.

Salutation to GAṆESA !

Invocation. 1. Having saluted that God, who when called upon brings all undertakings to a successful issue, and also that Goddess, through whose benign favour the tongues of poets, gifted with a flow of words ever new and with elegance, sweetness and playfulness, sport in their mouths as in a place of recreation, as dancing-girls adorned with beauty disport themselves in the dance with elegance and with every variety of step, I proceed to indite this work on the Sphere. It has been freed from all error, and rendered intelligible to the lowest capacity.

Object of the work. 2. Inasmuch as no calculator can hope to acquire in the assemblage of the learned a distinguished reputation as an Astronomer, without a clear understanding of the principles upon which all the calculations of the mean and other places of the planets are founded, and to remove the doubts which may arise in his own mind, I therefore proceed to treat of the sphere, in such a manner as to make the reasons of all my calculations manifest. On inspecting the Globe they become clear and manifest as if submitted to the eye, and are as completely at command, as the wild apple (áṇwlá) held in the palm of the hand.

Ridicule of an ignorance of the Spheric.

3. As a feast with abundance of all things but without clarified butter, and as a kingdom without a king, and an assemblage without eloquent speakers have little to recommend them; so the Astronomer who has no knowledge of the spheric, commands no consideration.

4. As a foolish impudent disputant, who ignorant of grammar (rudely) enters into the company of the learned and vainly prates, is brought to ridicule, and put to shame by the frowns and ironical remarks of even children of any smartness, so he, who is ignorant of the spheric, is exposed in an assemblage of the Astronomers, by the various questions of really accomplished Astronomers.

Object of the Armillary sphere.

5. The Armillary sphere is said, by the wise, to be a representation of the celestial sphere, for the purpose of ascertaining the proofs of the positions of the Earth, the stars, and the planets: this is a species of figure, and hence it is deemed by the wise to be an object of mathematical calculation.

In praise of mathematics.

6. It is said by ancient astronomers that the purpose of the science is judicial astrology, and this indeed depends upon the influence of the horoscope, and this on the true places of the planets: these (true places) can be found only by a perfect knowledge of the spheric. A knowledge of the spheric is not to be attained without mathematical calculation. How then can a man, ignorant of mathematics, comprehend the doctrine of the sphere &c.?

Who is likely to undertake the study with effect.

7. Mathematical calculations are of two kinds, Arithmetical and Algebraical: he who has mastered both forms, is qualified if he have previously acquired (a perfect knowledge of) the Grammar (of the Sanskrit Language,) to undertake the study of the various branches of Astronomy. Otherwise he may acquire the name (but never the substantial knowledge) of an Astronomer.

In praise of Grammar.

8. He who has acquired a perfect knowledge of Grammar, which has been termed VEDAVADANA i. e. the mouth of the VEDAS and domicile of SARASWATI, may acquire a knowledge of every other science—nay of the VEDAS themselves. For this reason it is that none, but he who has acquired a thorough knowledge of Grammar, is qualified to undertake the study of other sciences.

The opinion of others on this work, quoted with a view of extending the study of it.

9. O learned man; if you intend to study the spheric, study the Treatise of BHÁSKARA, it is neither too concise nor idly diffuse: it contains every essential principle of the science, and is of easy comprehension; it is moreover written in an eloquent style, is made interesting with questions; it imparts to all who study it that manner of correct expression in learned assemblages, approved of by accomplished scholars.

End of Chapter I.

CHAPTER II.

Questions on the General view of the Sphere.

Questions regarding the Earth.

1. This Earth being encircled by the revolving planets, remains stationary in the heavens, within the orbits of all the revolving fixed stars; tell me by whom or by what is it supported, that it falls not downwards (in space)?

2. Tell me also, after a full examination of all the various opinions on the subject, its figure and magnitude, how its principal islands mountains and seas are situated in it?

Questions regarding those calculations used in ascertaining planets' true places and their causes.

3. Tell me, O my father, why the place of a planet found out from well calculated AHARGAṆA (or enumeration of mean terrestrial days, elapsed from

the commencement of the KALPA)* by applying the rule of pro-

* [A KALPA is that portion of time, which intervenes between one conjunction of all the planets at the Horizon of LANKÁ (that place at the terrestrial equator, where the longitude is 76° E., reckoned from Greenwich) at the first point of Aries, and a subsequent similar conjunction. A KALPA consists of 14 MANUS and their 15 SANDHIS; each MANU lying between 2 SANDHIS. Each MANU contains 71 YUGAS; each YUGA is divided into 4 YUGÁNGHRIS viz., KRITA, TRETA', DWÁPARA and KALI, the length of each of these is as the numbers 4, 3, 2 and 1. The beginning and end of each YUGA'NGHRIS being each one 12th part of it are respectively called its SANDHYÁ and SANDHYA'NSA. The number of sidereal years contained in each YUGA'NGHRI, &c. are shown below;

KALI,	432,000,
DWA'PARA,	864,000,
TRETA,	1,296,000,
KRITA,	1,728,000,
YUGA,	4,320,000,
71 × YUGA = MANU,	4,294,080,000,
14 MANU,	306,720,000,
15 MANU SANDHIS each equal to a KRITA YUGÁNGHRI,	25,920,000,
KALPA,	4,320,000,000,

Of the present KALPA 6 MANUS with their 7 SANDHIS, 27 YUGAS and their three YUGA'NGHRI i. e. KRITA, TRETA, and DWA'PARA, and 3179 sidereal years of the fourth YUGA'NGHRI of the 28th YUGA of the 7th MANU, that is to say, 1,972,947,179 sidereal years have elapsed from the beginning of the present KALPA to the commencement of the SA'LIWA'HANA era. Now we can easily find out the number of years that have elapsed from the beginning of the present KALPA to any time we like.

By astronomical observations the number of terrestrial and synodic lunar days in any given number of years can be ascertained and then, with the result found, their number in a KALPA or YUGA can be calculated by the rule of proportion.

By this method ancient Astronomers found out the number of lunar and terrestrial days in a KALPA as given below.

1,602,999,000,000 (synodic) lunar days }
and 1,577,916,450,000 terrestrial days } in a KALPA.

With the foregoing results and a knowledge of the number of sidereal years contained in a KALPA as well of those that have passed, we can find out the number of mean terrestrial days from the beginning of a KALPA to any given day. This number is called AHARGANA and the method of finding it is given in GAṆITÁDHYÁYA by BHA'SKARA'CHA'RYA.

By the daily mean motions of the planets, ascertained by astronomical observations, the numbers of their revolutions in a KALPA are known and are given in works on Astronomy.

To find the place of a planet by the number of its revolutions, the number of days contained in a KALPA and the AHARGAṆA to a given day, the following proportion is used.

As the terrestrial days in a KALPA,
: the number of revolutions of a planet in a KALPA
: the AHARGAṆA :
: the number of revolutions and signs &c. of the planet in the AHARGAṆA.

By leaving out the number of revolutions, contained in the result found, the remaining signs &c. indicate the place of the planet.

Now, the intention of the querist is this, why should not this be the true place of a planet? In the GAṆITÁDHYÁYA. BHÁSKARÁCHA'RYA has stated the revolutions in a KALPA, but he has here mentioned the revolutions in a YUGA on account of his constant study of the S'ISHYA-DHÍVRÍDDHIDA-TANTRA, a Treatise on Astronomy by LALLA who has stated in it the revolutions in a YUGA.—B. D.]

portion to the revolutions in the YUGA* &c. is not a true one? (i. e. why is it only a mean and not the true place) and why the rules for finding the true places of the different planets are not of the same kind? What are the DESÁNTARA, UDAYÁNTARA, BHUJÁNTARA, and CHARA corrections?† What is the MANDOCHCHA‡ (slow or 1st Apogee) and S'ÍGHROCHCHA§ (quick or 2nd Apogee)? What is the node?

4. What is the KENDRA|| and that which arises from it (i. e. the sine, cosine, &c. of it)? What is the MANDAPHALA|| (the first equation) and S'ÍGHRAPHALA¶ (the 2nd equation) which depend on the sine of the KENDRA? Why does the place of a planet become true, when the MANDAPHALA or S'ÍGHRAPHALA

* [It may be proper to give notes explaining concisely the technical terms occurring in these questions, which have no corresponding terms in English, in order that the English Astronomer may at once apprehend these questions without waiting for the explanation of them which the Author gives in the sequel.—B. D.]

† [To find the place of a planet at the time of sun-rise at a given place, the several important corrections, i. e. the UDAYA'NTARA, BHUJA'NTARA, DES'ÁNTARA, and CHARA are to be applied to the mean place of the planet found out from the AHARGAṆA by the fact of the mean place being found from the AHARGAṆA for the time when a fictitious body, which is supposed to move uniformly in the Equinoctial, and to perform a complete revolution in the same time as the Sun, reaches the horizon of LANKA'. We now proceed to explain the corrections.

The UDAYA'NTARA and BHUJA'NTARA corrections are to be applied to the mean place of a planet found from the AHARGAṆA for finding the place of the planet at the true time when the Sun comes to the horizon of LANKA' arising from those two portions of the equation of time respectively, one due to the inclination of the ecliptic to the equinoctial and the other to the unequal motion of the Sun in the ecliptic.

The DES'A'NTARA and CHARA corrections are to be applied to the mean place of a planet applied with the UDAYA'NTARA and BHUJA'NTARA corrections, for finding the place of the planet at the time of sun rise at a given place.

The DES'A'NTARA correction due to the longitude of the place reckoned from the meridian of LANKA' and the CHARA correction to the ascentional difference. B. D.]

‡ [MANDOCHCHA is equivalent to the higher Apsis. The Sun's and Moon's MANDOCHCHAS (higher Apsides) are the same as their Apogees, while the other planets' MANDOCHCHAS are equivalent to their Aphelions. B. D.]

§ [S'I'GHROCHCHA is that point of the orbit of each of the primary planets (i. e. Mars, Mercury, Jupiter, Venus and Saturn) which is furthest from the Earth. B. D.]

|| [KENDRA is of two kinds, one called MANDA-KENDRA corresponds with the anomaly and the other called S'I'GHRA-KENDṚA is equivalent to the commutation added to or subtracted from 180° as the SIGRA-KENDRA is greater or less than 180° B. D.]

¶ [MANDA-PHALA is the same as the equation of the centre of a planet and S'I'GHRA-PHALA is equivalent to the annual parallax of the superior planet; and the elongation of the inferior planets. B. D.]

are (at one time) added to and (at another) subtracted from it? What is the twofold correction called DṚIKKARMA* which learned astronomers have applied (to the true place of a planet) at the rising and setting of the planet? Answer me all these questions plainly, if you have a thorough knowledge of the sphere.

Questions regarding the length of the day and night.

5. Tell me, O you acute astronomer, why, when the Sun is on the northern hemisphere, is the day long and the night short, and the day short and the night long when the Sun is on the southern hemisphere?

Questions regarding the length of the day and night of the Gods DAITYAS, PITṚIS and BRAHMA'.

6. How is it that the day and night of the Gods and their enemies DAITYAS correspond in length with the solar years? How is it that the night and day of the PITṚIS is equal in length to a (synodic) lunar month, and how is it that the day and night of BRAHMÁ is 2000 YUGAS† in length?

Questions regarding the periods of risings of the signs of the Zodiac.

7. Why, O Astronomer, is it that the 12 signs of the Zodiac which are all of equal length, rise in unequal times (even at the Equator,) and why are not those periods of rising the same in all countries?

Questions as to the places of the DYUJYÁ, the KUJYA', &c.

8. Shew me, O learned one, the places of the DYUJYÁ (the radius of the diurnal circle), the KUJYÁ (the sine of that part of the arc of the diurnal circle intercepted between the horizon and the six o'clock line, i. e. of the ascensional difference in terms

* [DṚIKKARMA is the correction requisite to be applied to the place of a planet, for finding the point of the ecliptic on the horizon when the planet reaches it. This correction is to be applied to the place of a planet by means of its two portions, one called the ÁYANA-DṚIKKARMA and the other the ÁKSHA-DṚIKKARMA. The place of a planet with the ÁYANA-DRIKKARMA applied, gives the point of the ecliptic on the six o'clock line when the planet arrives at it; and this corrected place of the planet, again with the ÁKSHA-DṚIKKARMA applied, gives the point of the ecliptic on the horizon when the planet comes to it. B. D.]

† The KṚITA, TRETA', DWA'PARA and KALI are usually called YUGAS: but the four together form only one YUGA, according to the SIDDHA'NTA system, each of these four being held to be individually but a YUGA'NGHRI. L. W.

of a small circle), and show me also the places of the declination, SAMA-SÁNKU,* AGRA (the sine of amplitude), latitude and co-latitude &c. in this Armillary sphere as these places are in the heavens.

Questions regarding certain differences in the times and places of solar and lunar Eclipses.

If the middle of a lunar Eclipse takes place at the end of the TITHI (at the full moon), why does not the middle of the solar Eclipse take place in like manner at the change? Why is the Eastern limb of the Moon in a lunar Eclipse first involved in obscurity, and the western limb of the Sun first eclipsed in a solar Eclipse ?†

Questions regarding the parallaxes.

9. What, O most intelligent one, is the LAMBANA‡ and what is the NATI? why is the LAMBANA applied to the TITHI and the NATI applied to the latitude (of the Moon)? and why are these corrections settled by means (of the radius) of the Earth?

Questions regarding the phases of the Moon.

10. Ah! why, after being full, does the Moon, having lost her pure brightness, lose her circularity, as it were, by her too close association, caused by her diurnal revolution with the night: and why again after having arrived in the same sign as the Sun, does she thenceforth, by successive augmentation of her pure

* [SAMA-SA'NKU is the sine of the Sun's altitude when it comes to the prime vertical. B. D.]

† [An Eclipse of the Moon is caused by her entering into the Earth's shadow and as the place of the Earth's shadow and that of the Moon is the same at the full moon, the conjunction of the Earth's shadow and the Moon must happen at the same time; and an Eclipse of the Sun is caused by the interposition of the Moon between the Earth and the Sun, and the conjunction of the Sun and Moon in like manner must happen at the new moon, as then the place of the Sun and Moon is the same. As this is the case with the eclipses of both of them (i. e. both the Sun and Moon) the querist asks, "If the middle of a lunar eclipse &c." It is scarcely necessary to add that the assumption that the middle of a lunar eclipse takes place exactly at the full moon, is only approximately correct. B. D.]

‡ [The LAMBANA is equivalent to the Moon's parallax in longitude from the Sun reduced into time by means of the Moon's motion from the Sun: and the NATI is the same as the Moon's parallax in latitude from the Sun. B. D.]

brightness, as from association with the Sun, attain her circular form ?*

End of the second Chapter.

CHAPTER III.

Called Bhuvana-kos'a or Cosmography.

The excellence of the Supreme Being.

1. The Supreme Being PARA BRAHMA the first principle, excels eternally. From the soul (PURUSHA) and nature (PRAKRITI,) when excited by the first principle, arose the first Great Intelligence called the MAHATTATTWA or BUDDHITATTWA : from it sprung self-consciousness (AHANKÁRA :) from it were produced the Ether, Air, Fire, Water, and Earth ; and by the combination of these was made the universe BRAHMÁNḌA, in the centre of which is the Earth : and from BRAHMÁ CHATURÁNANA, residing on the surface of the Earth, sprung all animate and inanimate things.

Description of the Earth.

2. This Globe of the Earth formed of (the five elementary principles) Earth, Air, Water, the Ether, and Fire, is perfectly round, and encompassed by the orbits of the Moon, Mercury, Venus, the Sun, Mars, Jupiter, and Saturn, and by the constellations. It has no (material) supporter ; but stands firmly in the expanse of heaven by its own inherent force. On its surface throughout subsist (in security) all animate and inanimate objects, DANUJAS and human beings, Gods and DAITYAS.

* This verse has a double meaning, all the native writers, however grave the subject, being much addicted to conceits. The second interpretation of this verse is as follows :

Ah ! why does the most learned of Brahmans, though distinguished by his immaculate conduct, lose his pure honour and influence as it were from his misconduct caused by derangement ? It is no wonder that the said Brahman after having met with a Brahman skilled in the VEDAS, and by having recourse to him, thenceforth becomes distinguished for his eminent good conduct by gradual augmentation of his illustriousness. L. W.

3. It is covered on all sides with multitudes of mountains, groves, towns and sacred edifices, as is the bulb of the Nauclea's globular flower with its multitude of anthers.

Refutation of the supposition that the Earth has successive supporters.

4. If the Earth were supported by any material substance or living creature, then that would require a second supporter, and for that second a third would be required. Here we have the absurdity of an interminable series. If the last of the series be supposed to remain firm by its own inherent power, then why may not the same power be supposed to exist in the first, that is in the Earth? For is not the Earth one of the forms of the eight-fold divinity i. e. of S'IVA.

Refutation of the objection, as to how the Earth has its own inherent power.

5. As heat is an inherent property of the Sun and of Fire, as cold of the Moon, fluidity of water, and hardness of stones, and as the Air is volatile, so the earth is naturally immoveable. For oh! the properties existing in things are wonderful.

6. The* property of attraction is inherent in the Earth. By this property the Earth attacts any unsupported heavy thing towards it: The thing appears to be falling [but it is in a state of being drawn to the Earth]. The etherial expanse being equally outspread all around, where can the Earth fall?

Opinion of the BAUDDHAS.

7. Observing the revolution of the constellations, the BAUDDHAS thought that the Earth had no support, and as no heavy body is seen stationary in the air, they asserted that the earth† goes eternally downwards in space.

Opinion of the JAINAS.

8. The JAINAS and others maintain that there are two Suns and two

* [It is manifest from this that neither can the Earth by any means fall downwards, nor the men situated at the distances of a fourth part of the circumference from us or in the opposite hemisphere. B. D.]

† [He who resides on the Earth, is not conscious of the motion of it downwards in space, as a man sitting on a moving ship does not perceive its motion, B. D.]

Moons, and also two sets of constellations, which rise in constant alternation. To them I give this appropriate answer.

Refutation of the opinion of the BAUDDHAS.

9. Observing as you do, O BAUDDHA, that every heavy body projected into the air, comes back again to, and overtakes the Earth, how then can you idly maintain that the Earth is falling down in space? [If true, the Earth being the heavier body, would, he imagines*—perpetually gain on the higher projectile and never allow its overtaking it.]

Refutation of the opinion of the JAINAS.

10. But what shall I say to thy folly, O JAINA, who without object or use supposest a double set of constellations, two Suns and two Moons? Dost thou not see that the visible circumpolar constellations take a whole day to complete their revolutions?

Refutation of the supposition that the Earth is level.

11. If this blessed Earth were level, like a plane mirror, then why is not the sun, revolving above at a distance from the Earth, visible to men as well as to the Gods? (on the PAURÁṆIKA hypothesis, that it is always revolving about MERU, above and horizontally to the Earth.

12. If the Golden mountain (MERU) is the cause of night, then why is it not visible when it intervenes between us and the Sun? And MERU being admitted (by the PAURÁṆIKAS) to lie to the North, how comes it to pass that the Sun rises (for half the year) to the South?

Reason of the false appearance of the plane form of the Earth.

13. As the one-hundredth part of the circumference of a circle is (scarcely different from) a plane, and as the Earth is an excessively large body, and a man exceedingly small (in comparison,) the whole visible portion of the Earth consequently appears to a man on its surface to be perfectly plane,

* [This was BHASKARA'S own notion;—but even on the more correct principle, that all bodies fall with equal rapidity, the argument holds good. B. D.]

Proof of the correctness of alleged circumference of the Earth.

14. That the correct dimensions of the circumference of the Earth have been stated may be proved by the simple Rule of proportion in this mode: (ascertain the difference in YUJANAS between two towns in an exact north and south line, and ascertain also the difference of the latitudes of those towns: then say) if the difference of latitude gives this distance in YUJANAS, what will the whole circumference of 360 degrees give?

To confirm the same circumference of the Earth.

15. As it is ascertained by calculation that the city of UJJAYINÍ is situated at a distance from the equator equal to the one-sixteenth part of the whole circumference: this distance, therefore, multiplied by 16 will be the measure of the Earth's circumference. What reason then is there in attributing (as the PAURÁṆIKAS do) such an immense magnitude to the earth?

16. For the position of the moon's cusps, the conjunction of the planets, eclipses, the time of the risings and settings of the planets, the lengths of the shadows of the gnomon, &c., are all consistent with this (estimate of the extent of the) circumference, and not with any other; therefore it is declared that the correctness of the aforesaid measurement of the earth is proved both directly and indirectly,—(directly, by its agreeing with the phenomena;—indirectly, by no other estimate agreeing with the phenomena).

17. LANKÁ is situated in the middle of the Earth: YAMAKOTI is situated to the East of LANKÁ, and ROMAKAPATTANA to the west. The city of SIDDHAPURA lies underneath LANKÁ. SUMERU is situated to the North (under the North Pole,) and VADAVÁNALA to the South of LANKÁ (under the south Pole):

18. These six places are situated at a distance of one-fourth part of the Earth's circumference each from its adjoining one. So those who have a knowledge of Geography maintain. At MERU reside the Gods and the SIDDHAS, whilst at VADAVÁNALA are situated all the hells and the DAITYAS.

19. A man on whatever part of the Globe he may be, thinks the Earth to be under his feet, and that he is standing up right upon it: but two individuals placed at 90° from each other, fancy each that the other is standing in a horizontal line, as it were at right angles to himself.

20. Those who are placed at the distance of half the Earth's circumference from each other are mutually antipodes, as a man on the bank of a river and his shadow reflected in the water: But as well those who are situated at the distance of 90° as those who are situated at that of 180° from you, maintain their positions without difficulty. They stand with the same ease as we do here in our position.

Positions of the Dwípas and Seas.

21. Most learned astronomers have stated that Jambúdwípa embraces the whole northern hemisphere lying to the north of the salt sea: and that the other six Dwípas and the (seven) Seas viz. those of salt, milk, &c. are all situated in the southern hemisphere.

22. To the south of the equator lies the salt sea, and to the south of it the sea of milk, whence sprung the nectar, the Moon and the Goddess Lakshmí, and where the Omnipresent Vásudeva, to whose Lotus-feet Brahmá and all the Gods bow in reverence, holds his favorite residence.

23. Beyond the sea of milk lie in succession the seas of curds, clarified butter, sugar-cane-juice, and wine: and, last of all, that of sweet Water, which surrounds Vadavánala. The Pátála Lokas or infernal regions, form the concave strata of the Earth.

24. In those lower regions dwell the race of serpents (who live) in the light shed by the rays issuing from the multitude of the brilliant jewels of their crests, together with the multitude of Asuras; and there the Siddhas enjoy themselves with the pleasing persons of beautiful females resembling the finest gold in purity.

25. The S'áka, S'álmala, Kaus'a, Kráuncha, Gomedaka, and

Pushkara Dwípas are situated [in the intervals of the above mentioned seas] in regular alternation: each Dwípa lying, it is said, between two of these seas.

Positions of the Mountains in Jambú dwípa and its nine Khaṇḍas parts caused by the mountains.

26. To the North of Lanká lies the Himálaya mountain, and beyond that the Hemakúṭa mountain and beyond that again the Nishadha mountain. These three Mountains stretch from sea to sea. In like manner to the north of Siddhapura lie in succession the S'ṛingaván S'ukla and Níla mountains. To the valleys lying between these mountains the wise have given the name of Varshas.

27. This valley which we inhabit is called the Bhárata-varsha; to the North of it lies the Kinnaravarsha, and beyond it again the Harivarsha, and know that the north of Siddhapura in like manner are situated the Kuru, Hiraṇmaya and Ramyaka Varshas.

28. To the north of Yamakoṭi lies the Málytaván mountain, and to the north of Romakapattana the Gandhamádana mountain. These two mountains are terminated by the Níla and Nishadha mountains, and the space between these two is called the Ilávṛita Varsha.

29. The country lying between the Málytaván mountain and the sea, is called the Bhadrás'wa-varsha by the learned; and geographers have denominated the country between the Gandhamádana and the sea, the Ketumála-varsha.

30. The Ilávṛita-varsha, which is bounded by the Nishadha, Níla, Gandhamádana and Malyaván mountains, is distinguished by a peculiar splendour. It is a land rendered brilliant by its shining gold, and thickly covered with the bowers of the immortal Gods.

Position of the mountain Meru in Ilávṛita.

31. In the middle of the Ilávṛita Varsha stands the mountain Meru, which is composed of gold and of precious stones, the abode of the immortal Gods. Expounders of the Puráṇas have further described this Meru to be the pericarp of the earth-lotus whence Brahmá had his birth.

32. The four mountains MANDARA, SUGANDHA, VIPULA and SUPÁRS'WA serve as buttresses to support this MERU, and upon these four hills grow severally the KADAMBA, JAMBÚ, VAṬA and PIPPALA trees which are as banners on those four hills.

33. From the clear juice which flows from the fruit of the JAMBÚ springs the JAMBÚ-NADÍ; from contact with this juice earth becomes gold: and it is from this fact that gold is called JÁMBÚNADA: [this juice is of so exquisite a flavour that] the multitude of the immortal Gods and SIDDHAS, turning with distaste from nectar, delight to quaff this delicious beverage.

34. And it is well known that upon those four hills [the buttresses of MERU] are four gardens, (1*st*) CHAITRARATHA of varied brilliancy [sacred to KUBERA], (2*nd*) NANDANA which is the delight of the APSARAS, (3*rd*) the DHṚITI which gives refreshment to the Gods, and (4*th*) the resplendent VAIBHRÁJA.

35. And in these gardens are beautified four reservoirs, viz. the ARUṆA, the MÁNASA, the MAHÁHRADA and the S'WETA-JALA, in due order: and these are the lakes in the waters of which the celestial spirits, when fatigued with their dalliance with the fair Goddesses, love to disport themselves.

36. MERU divided itself into three peaks, upon which are situated the three cities sacred to VISHṆU, BRAHMÁ and S'IVA [denominated VAIKUNTHA, BRAHMAPURA, and KAILÁSA], and beneath them are the eight cities sacred to INDRA, AGNI, YAMA, NAIRṚITA, VARUṆA, VÁYU, S'AS'Í, and Ís'a, [i. e. the regents of the eight DIKS or directions,* viz., the east sacred to

* [As the point where the equator cuts the horizon is the east, the sun therefore rises due east at time of the equinoxes but on this ground, we cannot determine the direction at MERU [the north pole] because there the equator coincides with the horizon and consequently the sun moves at MERU under the horizon the whole day of the equinox. Yet the ancient astronomers maintained that the direction in which the YAMAKOṬI lies from MERU is the east, because, according to their opinion, the inhabitants of MERU saw the sun rising towards the YAMAKOṬI at the beginning of the KALPA. In the same manner, the direction in which LANKÁ lies from mount MERU is south, that in which ROMAKAPATTANA lies, is west, and the direction in which SIDDHA-

Indra, the south-east sacred to Agni, the south sacred to Yama, the south-west sacred to Nairṛita, the west sacred to Varuṇa, the north-west sacred to Váyu, the north sacred to S'as'í and the north-east sacred to Ís'a.]

Some peculiarity.

37. The sacred Ganges, springing from the Foot of Vishnu, falls upon mount Meru, and thence separating itself into four streams descends through the heavens down upon the four Vishkambhas or buttress hills, and thus falls into the four reservoirs [above described].

38. [Of the four streams above mentioned], the first called Sítá, went to Bhadrás'wa-varsha, the second, called Alakanandá, to Bhárata-varsha, the third, called Chakshu, to Ketumála-varsha, and the fourth, called Bhadrá to Uttara Kuru [or North Kuru].

39. And this sacred river has so rare an efficacy that if her name be listened to, if she be sought to be seen, if seen, touched or bathed in, if her waters be tasted, if her name be uttered, or brought to mind, and her virtues be celebrated, she purifies in many ways thousands of sinful men [from their sins].

40. And if a man make a pilgrimage to this sacred stream, the whole line of his progenitors, bursting the bands [imposed on them by Yama], bound away in liberty, and dance with joy; nay even, by a man's approach to its banks they repulse the slaves of Yama [who kept guard over them], and, escaping from Naraka [the infernal regions], secure an abode in the happy regions of Heaven.

pura lies from Meru is north. The buttresses of Meru, Mandara, Sugandha, &c. are situated in the east, south &c. from Meru respectively. B. D.]

Note on verses from 21 to 43:—Bhaskara'cha'rya has exercised his ingenuity in giving a locality on the earth to the poetical imaginations of Vya'sa, at the same time that he has preserved his own principles in regard to the form and dimensions of the Earth. But he himself attached no credit to what he has described in these verses for he concludes his recital in his commentary with the words.

यदिदमुक्तं तत् सव पुराणाश्रितम् ।

"What is stated here rests all on the authority of the Puráṇas." As much as to say "credat Judæus." L. W.

The 9 KHANḌAS and 7 KULÁCHALAS of BHA'RATA-VARSHA.

41. Here in this BHÁRATA-VARSHA are embraced the following nine KHANḌAS [portions] viz. AINDRA, KAS'ERU, TÁMRAPARṆA, GABHASTIMAT, KUMARIKÁ, NÁGA, SAUMYA, VÁRUṆA, and lastly GÁNDHARVA.

42. In the KUMÁRIKÁ alone is found the subdivision of men into castes; in the remaining KHANḌAS are found all the tribes of ANTYAJAS or outcaste tribes of men. In this region [BHÁRATA-VARSHA] are also seven KULÁCHALAS, viz. the MÁHENDRA, S'UKTI, MALAYA, ṚIKSHAKA, PÁRIYÁTRA, the SAHYA, and VINDHYA hills.

Arrangement of the seven LOKAS worlds.

43. The country to the south of the equator is called the BHÚRLOKA, that to the north the BHUVALOKA and MERU [the third] is called the SWARLOKA, next is the MAHARLOKA in the Heavens beyond this is the JANALOKA, then the TAPOLOKA and last of all the SATYALOKA. These LOKAS are gradually attained by increasing religious merits.

44. When it is sunrise at LANKÁ, it is then midday at YAMAKOṬI (90° east of LANKÁ), sunset at SIDDHAPURA and midnight at ROMAKAPATTANA.

Points of the compass why MERU is due north of all places.

45. Assume the point of the horizon at which the sun rises as the east point, and that at which he sets as the west point, and then determine the other two points, i. e., the north and south through the MATSYA* effected by the east and west points. The line connecting the north and south points will be a meridian line and this line in whatever place it is drawn will fall upon the north point: hence MERU lies due north of all places.

A curious fact is rehearsed. Geographical Anomaly.

46. Only YAMAKOṬI lies due east from UJJAYINÍ, at the distance of 90°

* [From the east and west points, as centres, with a common radius describe two arcs, intersecting each other in two points, the place contained by the arcs is called MATSYA "a fish" and the intersecting points are the north and south points. B. D.]

from it: but LANKÁ and not UJJAYINÍ lies due west from YAMAKOṬI.

47. The same is the case everywhere; no place can lie west of that which is to its east except on the equator, so that east and west are strangely related.*

Right sphere.

48. A man situated on the equator sees both the north and south poles touching [the north and south points of] the horizon, and the celestial sphere resting (as it were) upon the two poles as centres of motion and revolving vertically over his head in the heavens, as the Persian water-wheel.

Oblique sphere.

49. As a man proceeds north from the equator, he observes the constellations [that revolve vertically over his head when seen from the equator] to revolve obliquely, being deflected from his vertical point: and the north pole elevated above his horizon. The degrees between the pole and the horizon are the degrees of latitude [at the place]. These degrees are caused by the YOJANAS [between the equator and the place].

How the degrees of latitude are produced from the distance in YOJANAS and vice versâ.

50. The number of YOJANAS [in the arc of any terrestrial or celestial circle] multiplied by 360 and divided by [the number in YOJANAS in] the circumference of the circle is the number of degrees [of that arc] in the earth or in the planetary orbit in the heavens. The YOJANAS are found from the degrees by reversing the calculation.

Parallel sphere.

51. The Gods who live in the mount MERU observe at their zenith

[* As the sun or any heavenly body when it reaches the Prime Vertical of any place is called due east or west, so according to the Hindu Astronomical language all the places on the Earth which are situated on the circle corresponding to the Prime Vertical are due east or west from the place and not those which are situated on the parallel of latitude of the place, that is the places which have the angle of position 90° from any place are due east or west from that place. And thus all directions on the Earth are shown by means of the angle of position in the Hindu Astronomical works. B. D.]

the north pole, while the DAITYAS in VADAVÁNALA the south pole. But while the Gods behold the constellations revolving from left to right, to the DAITYAS they appear to revolve from right to left. But to both Gods and DAITYAS the equatorial constellations appear to revolve on and correspond with the horizon.

Dimensions of the Earth's circumference.

52. The circumference of the earth has been pronounced to be 4967 YOJANAS and the diameter of the same has been declared to be $1581\frac{1}{24}$ YOJANAS in length: the superficial area of the Earth, like the net enclosing the hand ball, is 78,53,034 square YOJANAS, and is found by multiplying the circumference by the diameter.*

The error of Lalla is exposed in regard to the superficial area of the Earth.

53. The superficial area of the Earth, like the net enclosing the hand ball, is most erroneously stated by LALLA: the true area not amounting to one hundredth part of that so idly assumed by him. His dimensions are contrary to what is found by actual inspection: my charge of error therefore cannot be pronounced to be rude and uncalled for. But if any doubt be entertained, I beg you, O learned mathematicians, to examine well and with the utmost impartiality whether the amount stated by me or that stated by him is the correct one. [The amount stated by LALLA in his

* [The diameter and the circumference of the Earth here mentioned are to each other as 1250: 3927 and the demonstration of this ratio is shown by BHÁSKARÁCHÁRYA in the following manner.

Take a radius equal to any large number, such as more than 10000, and through this determine the sine of a smaller arc than even the 100th part of the circumference of the circle by the aid of the canon of sines (JYOTPATTI,) and the sine thus determined when multiplied by that number which represents the part which the arc just taken is of the circumference, becomes the length of circumference because an arc smaller than the 100th part of the circumference of a circle is [scarcely different from] a straight line. For this reason, the circumference equal to the number 62832 is granted by ARYABHATTA and the others, in the diameter equal to the number 20,000. Though the length of the circumference determined by extracting the square root of the tenfold square of the diameter is rough, yet it is granted for convenience by SRIDHARÁCHÁRYA, BRAHMAGUPTA and the others, and it is not to be supposed that they were ignorant of this roughness.—B. D.]

work entitled DHÍVṚIDDHIDA-TANTRA is 285,63,38,557 square YOJANAS, which he appears to have found by multiplying the square contents of the circle by the circumference.]

Shows the wrongness of the Rule given by Lalla.

54. If a piece of cloth be cut in a circular form with a diameter equal to half the circumference of the sphere, then half of the sphere will be (entirely) covered by that circular cloth and there will still be some cloth to spare.

55. As the area of this piece of cloth is to be found nearly 2¼ times the area of a great circle of the sphere: and the area of the piece of cloth covering the other half of the sphere is also the same; *

56. Therefore the area of the whole sphere cannot be more than 5 times the area of the great circle of the sphere. How then has he multiplied [the area of the great circle of the sphere] by the circumference [to get the superficial contents of the sphere] ?

57. As the area of a great circle [of the sphere] multiplied by the circumference is without reason, the rule (therefore of LALLA for the superficial contents of the sphere) is wrong, and the superficial area of the Earth (given by him) is consequently wrong.

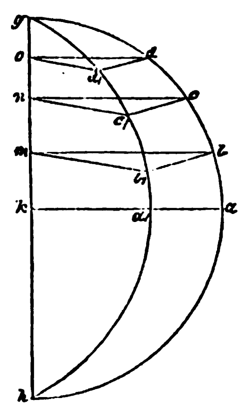
Otherwise.

58, 59. Suppose the length of the [equatorial] circumference of the globe equal to 4 times the number of sines [viz. 96, there being 24 sines calculated for every 3°¾, which number multiplied by 4 = 96] and such oblong sections equal to the number of the length of the said circumference and marked with the vertical lines [running from pole to pole], as there are seen formed by nature on the ÁNWLÁ fruit marked off by the lines running from the top of it to its bottom.

* Let the diameter of a sphere be 7: the circumference will be 22 nearly. The area of a circle whose diameter is 7 will be about 38½; that of a circle whose diameter is 11 (½ circumference) will be about 89[illegible] this 89[illegible] is little less than 2¼ times 38½. L. W.

60. If we determine the superficial area of one of these sections by means of its parts, we have it in this form. Sum of all the sines diminished by half of the radius and divided by the same.*

* The correctness of this form is thus briefly illustrated by **Bha'skara'cha'rya** in his commentary.



Let $g\,a\,h\,a_1\,g$ be the section in which ab, $b\,c$, $c\,d$ &c. and a_1b_1, b_1c_1, c_1d_1, &c. are each equal to 1 cubit and also aa_1 are equal to 1 cubit: then bb_1, cc_1, dd_1, &c. are proportional to the sines $m\,b$, $n\,c$, $o\,d$, &c. and are thus found.

If ka or rad : give, aa 1 (= 1) : : mb : $bb_1 = \frac{mb.}{Rad.}$

If Rad : 1 : : nc : $cc_1 = \frac{nc}{Rad}$

again Rad : 1 : : od : $dd_1 = \frac{od}{Rad}$

&c.

Now aa_1, bb_1, cc_1, &c. being found, the contents of each of $aa_1\,b_1b$, $bb_1\,c_1c$, $cc_1\,d_1d$, &c. the part of the section is found by taking half the sum of aa_1 & bb_1, bb_1 & cc_1, cc_1 & dd_1 &c. and multiplying it by ab (which is equal to each of bc, cd, &c.) here ab is assumed as 1 and the whole surface each of aa_1b_1b, bb_1c_1c as a plane, for an arc of $3°\frac{3}{4}$ is scarcely different from a plane.

Now to find the sum of aa_1b_1b, bb_1c_1c &c. we have

$$\frac{aa_1+bb_1}{2}\times 1+\frac{bb_1+cc_1}{2}\times 1+\frac{cc_1+dd_1}{2}\times 1+\&c.$$

adding these and leaving out 1 multiplier, we have

$$\tfrac{1}{2}\,aa_1+bb_1+cc_1+dd_1+\&c.$$

Substituting the values of aa_1, bb_1, &c. we have

$\frac{1}{2}+\frac{mb}{R}+\frac{nc}{R}+\frac{od}{R}+$ &c. so on for the assumed sines

but $\frac{1}{2}=\frac{\frac{1}{2}R}{R}=\frac{R}{R}-\frac{\frac{1}{2}R}{R}$

By substitution we get

$$\frac{R}{R}+\frac{mb}{R}+\frac{nc}{R}+\&c.\ldots-\frac{\frac{1}{2}R}{R}$$

$$=\frac{R+mb+nc+od+\&c\ldots-\frac{1}{2}R}{R}$$

It is evident from this that the sum of all the sines diminished by the half of the Radius and divided by the Radius is equal to the contents of the upper half of the section, therefore by dividing by $\frac{1}{2}$ Rad we get the whole section instead of only the upper half of it.

i. e. contents of the whole section $=\dfrac{\text{sum of all the sines}-\frac{1}{2}R.}{\frac{1}{2}R}=A.$

61. As the superficial area of one section thus determined is equal to the diameter of the globe, the product found by multiplying the diameter by the circumference has therefore been asserted to be the superficial contents of a sphere.

The grand deluges or dissolutions.

62. The earth is said to swell to the extent of one YOJANA equally all around [from the centre] in a day of BRAHMÁ by reason of the decay of the natural productions which grow upon it: in the BRÁHMA deluge that increase is again lost. In the grand deluge [in which BRAHMÁ himself as well as all nature fades away then] the Earth itself is reduced to a state of nonentity.

Are four-fold.

63. That extinction which is daily taking place amongst created beings is called the DAINANDINA or daily extinction. The BRÁHMA extinction or deluge takes place at the end of BRAHMÁ's day: for all created beings are then absorbed in Brahmá's body.

64. As on the extinction of BRAHMÁ himself all things are dissolved into nature, wise men therefore call that dissolution the PRÁKṚITIKA or resolution into nature. Things thus in a state of extinction having their destinies severally fixed are again produced in separate forms when nature is excited (by the Creator).

65. The devout men, who have destroyed all their virtues and sins by a knowledge of the soul, having abstracted their minds from worldly acts, concentrate their thoughts on the

Here, by substituting the values of the 24 sines stated in the GAṆITA'DHA'YA we have

$A = 30\frac{13}{23}$ viz. the diameter of the globe where the circumference $= 96$. L. W.

[Here, the demonstration of the rule (multiply the superficial area of the sphere by the diameter and divide the product by 6) for finding out the solid content of the sphere is shown by BHA'SKARA'CHA'RYA in the following manner.

Suppose in the sphere the number of pyramids, the height of which is equal to the radius and whose bases are squares having sides equal to 1, equal to the number of the superficial area of the sphere, then

The solid contents of every pyramid $= \frac{1}{3}$ R.

$= \frac{1}{6}$ diameter

and the number of pyramids in the sphere is equal to the number of the superficial contents of the sphere.

$\therefore$ The solid content of the sphere $= \frac{1}{6}$ diameter $\times$ superficial area.—B.D.]

Supreme Being, and after their death, as they attain the state from which there is no return, the wise men therefore denominate this state the ÁTYANTIKA dissolution. Thus the dissolutions are four-fold.

The universe.

66. The earth and its mountains, the Gods and DÁNAVAS, men and others and also the orbits of the constellations and planets and the LOKAS which, it is said, are arranged one above the other, are all included in what has been denominated the BRAHMÁNḌA (universe).

Dimensions of the BRAHMA'NḌA.

67. Some astronomers have asserted the circumference of the circle of Heaven to be 18,712,069,200,000,000 YOJANAS in length. Some say that this is the length of the zone which binds the two hemispheres of the BRAHMÁNḌA. Some PAURÁṆIKAS say that this is the length of the circumference of the LOKÁLOKA PARVATA.*

* Vide verses 67,68,69, BHA'SKARA'CHA'RYA does not answer the objection which these verses supply to his theory of the Earth being the centre of the system. The Sun is here made the principal object of the system—the centre of the BRAHMA'NḌA—the centre of light whose boundary is supposed fixed: but if the Sun moves then the Hindoo BRAHMA'NḌA must be supposed to be constantly changing its Boundaries. Subbuji Bápú had not failed to use this argument in favour of the Newtonian system in his S'IROMAṆI PRAKA'S'A, vide pages 55, 56. BHA'SKARA'CHA'RYA however denies that he can father the opinion that this is the length of the circumference limiting the BRAHMA'NḌA and thus saves himself from a difficulty. L. W.

[Mr. Wilkinson has thus shown the objection which Subbáji Bápú made to the assumption of the Sun's motion, but I think that the objection is not a judicious one. Because had the length of the circumference of the BRAHMA'NḌA been changed on account of the alteration of the boundary of the Sun's light with him, or had any sort of motion of the stars been assumed, as would have been granted if the earth is supposed to be fixed, then, the inconvenience would have occurred; but this is not the case. In fact, as we cannot fix any boundary of the light which issued from the sun, the stated length of the circumference of the BRAHMÁNḌA is an imaginary one. For this reason, BHÁSKARÁCHÁRYA does not admit this stated length of the circumference of the BRAHMA'NḌA. He stated in his GAṆITA'DHYAYA' in the commentary on the verse 68th of this Chapter that "those only, who have a perfect knowledge of the BRAHMA'NḌA as they have of an A'NVALA' fruit held in their palm, can say that this length of the circumference of the BRAHMA'NḌA is the true one;" that is, as it is not in man's power to fix any limit of the BRÁHMA'NḌA, the said limit is unreasonable. Therefore no objection can be possibly made to the system that the Sun moves, by assuming such an imaginary limit of the BRAHMÁNḌA which is little less impossible than the existence of the heavenly lotus.—B. D.]

68. Those, however, who have had a most perfect mastery of the clear doctrine of the sphere, have declared that this is the length of that circumference bounding the limits, to which the darkness dispelling rays of the Sun extend.

69. But let this be the length of the circumference of the BRAHMÁṆDA or not: [of that I have no sure knowledge] but it is my opinion that each planet traverses a distance corresponding to this number of YOJANAS in the course of a KALPA or a day of BRAHMÁ and that it has been called the KHAKAKSHÁ by the ancients.

End of third Chapter called the BHUVANA-KOS'A or cosmography.

CHAPTER IV.

CALLED MADHYA-GATI-VÁSANÁ.

On the principles of the Rules for finding the mean places of the Planets.

Places of the several winds.

1. The seven [grand] winds have thus been named: viz.—

1st. The Ávaha or atmosphere.
2nd. The Pravaha beyond it.
3rd. The Udvaha.
4th. The Samvaha.
5th. The Suvaha.
6th. The Parivaha.
7th. The Parávaha.

2. The atmosphere extends to the height of 12 YOJANAS from the Earth: within this limit are the clouds, lightning, &c. The Pravaha wind which is above the atmosphere moves constantly to the westward with uniform motion.

3. As this sphere of the universe includes the fixed stars and planets, it therefore being impelled by the Pravaha wind, is carried round with the stars and planets in a constant revolution.

An illustration of the motion of the planets.

4. The Planets moving eastward in the Heavens with a slow motion, appear as if fixed on account of the rapid motion of the sphere of the Heavens to the west, as insects moving reversely on a whirling potter's wheel appear to be stationary [by reason of their comparatively slow motion].

Sidereal and terrestrial days and their lengths.

5. If a star and the Sun rise simultaneously [on any day], the star will rise again (on the following morning) in 60 sidereal GHAṬIKÁS: the Sun, however, will rise later by the number of ASUS (sixths of a sidereal minute), found by dividing the product of the Sun's daily motion [in minutes] and the ASUS which the sign, in which the Sun is, takes in rising, by 1800 [the number of minutes which each sign of the ecliptic contains in itself].

6. The time thus found added to the 60 sidereal GHAṬIKÁS forms a true terrestrial day or natural day. The length of this day is variable, as it depends on the Sun's daily motion and on the time [which different signs of the ecliptic take] in rising, [in different latitudes: both of which are variable elements].*

* [Had the Sun moving with uniform motion on the equinoctial, the each minute of which rises in each ASU, the number of ASUS equal to the number of the minutes of the Sun's daily motion, being added to the 60 sidereal GHAṬIKAS, would have invariably made the exact length of the true terrestrial day as LALLA and others say. But this is not the case, because the Sun moves with unequal motion on the ecliptic, the equal portions of which do not rise in equal times on account of its being oblique to the equinoctional. Therefore, to find the exact length of the true terrestrial day, it is necessary to determine the time which the minutes of the Sun's daily motion take in rising and then add this time to 60 sidereal GHAṬIKA's. For this reason, the terrestrial day determined by LALLA and others is not a true but it is a mean.

The difference between the oblique ascension at the beginning of any given day, and that at the end of it or at the beginning of the next day, is the time which the minutes of the Sun's motion at the day above alluded to take in rising, but as this cannot be easily determined, the ancient Astronomers having determined the periods which the signs of the ecliptic take in rising at a given place, find the time which any portion of a given sign of the ecliptic takes in rising, by the following proportion.

If 30° or 1800′ of a sign: take number of the ASUS (which any given sign of the ecliptic takes) in rising at a given place : : what time will any portion of the sign above alluded to take in rising?

The calculation which is shown in the 5th verse depends on this proportion.—B. D.]

Revolutions of the Sun in a year are less than the revolutions of stars by one.

7. A sidereal day consists invariably of 60 sidereal GHAṬIKA'S: a mean SÁVANA day of the Sun or terrestrial day consists of that time with an addition of the number of ASUS equal to the number of the Sun's daily mean motion [in minutes]. Thus the number of terrestrial days in a year is less by one than the number of revolutions made by the fixed stars.

Length of solar year.

8. The length of the (solar) year is 365 days, 15 GHAṬIKÁS, 30 PALAS, 22½ VIPALAS reckoned in BHÚMI SA'VANA or terrestrial days: The $\frac{1}{12}$th of this is called a SAURA (solar) month, viz. 30 days, 26 GHAṬIKA'S, 17 PALAS, 31 VIPALAS, 52½ PRAVIPALAS. Thirty SÁVANA or terrestrial days make a SÁVANA month.*

Length of lunar month or lunation.

9. The time in which the Moon [after being in conjunction with the Sun] completing a revolution with the difference between the daily motion and that of the Sun, again overtakes the Sun, (which moves at a slower rate) is called a Lunar month. It is 29 days, 31 GHAṬIKA'S, 50 PALAS in length.†

The reason of additive months called ADHIMA'SAS.

10. An ADHIMÁSA or additive month which is lunar, occurs in the duration of 32½ SAURA (solar) months found by dividing the lunar month by the difference between this and the SAURA month. From

* [Here a solar year consists of 365 days, 15 GHAṬIKÁS, 30 PALAS, 22½ VIPALAS, i. e. 365 *d.* 6 *h.* 12 *m.* 9 *s.* and in SÚRYA-SIDDHA'NTA the length of the year is 365 *d.* 15 *g.* 31 *p.* 31. 4 *v.* i. e. 365 *d.* 6 *h.* 12 *m.* 36. 56 *s.*—B. D.]

[† That lunar month which ends, when the Sun is in MESHA stellar Aries is called CHAITRA and that which terminates when the Sun is in VṚISHABHA stellar Taurus, is called VAIS'AKHA and so on. Thus, the lunar months corresponding to the 12 stellar signs MESHA (Aries) VṚISHABHA (Taurus) MITHUNA (Gemini) KARKA (Cancer), SINHA (Leo), KANYA' (Virgo), TULA' (Libra), VRIS'CHIKA (Scorpio), DHANU (Sagittarius), MAKARA (Capricornus), KUMBHA (Aquarius) and MINA (Pisces), aro CHAITRA, VAIS'A'RKHA, JYESHṬHA, A'SHA'ḌHA S'RA'VAṆA, BHA'DRAPADA, A'S'WINA, KA'RTIKA, MA'RGAS'ÍRSHA, PAUSHA, MA'GHA, and PHALGUNA. If two lunar months terminate when the Sun is onl; in one stellar sign, the second of thcso is callod ADHIMA'SA an additivo month. The 30th part of a lunar month is called Tithi (a lunar day).—B. D.]

this, the number of the additive months in a KALPA may also be found by proportion.*

11. As a mean lunar month is shorter in length than a mean SAURA month, the lunar months are therefore more in number than the SAURA in a KALPA. The difference between the number of lunar and SAURA months in a KALPA is called by astronomers the number of ADHIMÁSAS in that period.

The reason of subtractive day called AVAMA.

12. An AVAMA or subtractive day which is SÁVANA occurs in $64\frac{1}{11}$ TITHIS (lunar days) found by dividing 30 by the difference between the lunar and SÁVANA month. From this, the number of AVAMAS in a YUGA may be found by proportion.†

13.‡ If the ADHIMÁSAS are found from SAURA days or months, then the result found is in the lunar months, [as for instance in finding the AHARYANA. If in the SAURA days of a KALPA : are

* [After the commencement of a YUGA, a lunar month terminates at the end of AMAVA'SYA' (new moon) and a SAURA month at the mean VRISHABHA-SANKRA'NTI (i. e. when the mean Sun enters the second stellar sign) which takes place with 54 *g.* 27 *p.* 31 *v.* 52½ *p.* after the new moon. Afterwards a second lunar month ends at the 2nd new moon after which the MITHUNA-SANKRA'NTI takes place with twice the Ghatis. &c. above mentioned. Thus the following SANKRA'NTIS KARKA &c. take place with thrice four times &c. those GHAṬIS, &c. In this manner, when the SANKRA'NTI thus going forward, again takes place at new moon, the number of the passed lunar months exceeds that of the SAURA by one. This one month is called an additive month : and the SAURA months which an additive month requires for its happening can be found by the proportion as follows.

As 54 *ghatis*, 27 p. &c. the difference between a lunar and a saura
: One saura month
: : 29, 31, 50 the number terrestrial day &c. in a lunar month
: 32, 15, 31, &c. the number of saura months, days, &c.—B. D.]

† [At the beginning of a KALPA or a YUGA, the terrestrial and lunar days begun simultaneously, but the lunar day being less than the terrestrial day, terminated before the end of the terrestrial day, i. e. before the next sun-rise. The interval between the end of the lunar day and the next sunrise, is called AVAMA-S'ESHA the remainder of the subtractive day. This remainder increases every day, therefore, when it is 60 GHAṬIKÁS (24 hours), this constitutes a AVAMA day or subtractive day. The lunar days in which a subtractive day occurs, are found by the following proportion.

If 0 *d.* 28 *g.* 10 *p.* the difference between the lengths of terrestrial and of a lunar month.
: 1 lunar month or 30 tithis
: : a whole terrestrial day : $64\frac{1}{11}$ tithis nearly.—B. D.]

‡ The objects of these two verses seems not to be more than to assert that the fourth term of a proportion is of the same denomination as the 2nd.—L. W.

so many ADHIMÁSAS : : then in given number of solar days; how many ADHIMÁSAS ?] If the ADHIMÁSAS are found from lunar days or months, then the result is in SAURA months, and the remainder is of the like denomination.

14. [In like manner] the AVAMAS or subtractive days if found from lunar days, are in SÁVANA time: if found from SÁVANA time they are lunar and the remainder is so likewise.

A question.

15. Why, O Astronomer, in finding the AHARGAṆA do you add SAURA months to the lunar months CHAITRA &c. [which may have elapsed from the commencement of the current year]: and tell me also why the [fractional] remainders of ADHIMÁSAS and AVAMA days are rejected: for you know that to give a true result in using the rule of proportion, remainders should be taken into account ?

Reason of omitting to include the ADHIMÁSA S'ESHA in finding the AHARGAṆA.

16.* As the lunar month ends at the change of the Moon and the SUARA month terminates when the Sun enters a stellar sign, the accumulating portion of an ADHIMÁSA always lies after each new Moon and before the Sun enters the sign.

* [The meaning of these 4 verses will be well understood by a knowledge of the rule for finding the AHARGAṆA, we therefore show the rule here.

In order to find the AHARGANA (elapsed terrestrial days from the commencement of the KALPA to the required time) astronomers multiply the number of SAURA years expired from the beginning of the KALPA by 12, and thus they get the number of SAURA months till the last MESHA SANKRANTI (that is, the time when the Sun enters the 1st sign of the Zodiac called Aries.) To these months they add then the passed lunar months CHAITRA &c., considering them as SAURA. These SAURA months become, up to the time when the Sun enters the sign of the Zodiac corresponding to the required lunar month. They multiply then the number of these months by 30 and add to this product the number of the passed TITHIS (lunar days) of the required month considering them as SAURA days. The number of SAURA days thus found becomes greater than that of those till the end of the required TITHI by the ADHIMASA S'ETHA. To make these SAURA days lunar, they determine the elapsed additive months by the proportion in the following manner

As the number of SAURA days in a KALPA
: the number of additive months in that period
: : the number of SAURA days just found
: the number of additive months elapsed

17. Now the number of TITHIS (lunar days) elapsed since the change of the Moon and supposed as if SAURA, is added to the number of SAURA days [found in finding the AHARGAṆA]: but as this number exceeds the proper amount by the quantity of the ADHIMÁSA-S'ESHA therefore the ADHIMÁS-SESHA is omitted [to be added].

18. [In the same manner] there is always a portion of a AVAMA-S'ESHA between the time of sun-rise and the end of the [preceding] TITHI. By omitting to subtract it, the AHARGAṆA is found at the time of sun-rise: if it were not omitted, the AHARGAṆA would represent the time of the end of the TITHI [which is not required but that of the sun-rise].

Reason of the correction called the UDAYÁNTARA KARMA.

19, 20, 21 and 22. As the true, terrestrial day is of variable length, the AHARGAṆA has been found in mean terrestrial days: the places of the planets found by this AHARGAṆA when rectified by the amount of the correction called the UDAYÁNTARA whether additive or subtractive will be found to be at the time of sun-rise at LANKÁ.* The ancient

If these additive months with their remainder be added to the SAURA days above found, the sum will be the number of lunar days to the end of the SAURA days, but we require it to the end of the required TITHI. And as the remainder of the additive months lies between the end of the TITHI and that of its corresponding SAURA days, they therefore add the whole number of ADHI-MÁSAS just found to that of the SAURA days omitting the remainder to find the lunar days to the end of the required TITHI. Moreover, to make these lunar days terrestrial, they determine AVAMA subtractive days by the proportion such as follows.

As the number of lunar days in a KALPA
: the number of subtractive days in that period
: : the number of lunar days just found
: the number of AVAMA elapsed with their remainder.

If these AVAMAS be subtracted with their remainder from the lunar days, the difference will be the number of the AVAMA days elapsed to the end of the required TITHI; but it is required at the time of sun-rise. And as the remainder of the subtractive days lies between the end of the TITHI and the sun-rise, they therefore subtract the AVAMAS above found from the number of lunar days omitting their remainder i. e. AVAMA-S'ESHA. Thus the AHARGAṆA itself becomes at the sun-rise.—B. D.]

* [If the Sun been moving on the equinoctial with an equal motion, the terrestrial day would have been of an invariable length and consequently the Sun would have reached the horizon at LANKÁ at the end of the AHARGAṆA which is an enumeration of the days of invariable length that is of the mean terrestrial days. But the Sun moves on the ecliptic whose equal parts do not

Astronomers have not thus rectified the places of the planets by this correction, as it is of a variable and small amount.

The difference between the number of ASUS of the right ascension of the mean Sun [found at the end of the AHARGAṆA] and the number of ASUS equal to the number of minutes of the mean longitude of the Sun [found at the same time] is the difference between the true and mean AHARGAṆAS.* Multiply this difference by the daily motion of the planet and divide the product by the number of ASUS in a nycthemeron.† The result [thus found] in minutes is to be subtracted from the places of the planets, if the ASUS [of the right ascension of the mean Sun] fall short of the KALÁS or minutes [of the mean longitude of the Sun], otherwise the result is to be added to the places of the planets. Instead of the right ascension, if oblique ascension be taken [in this calculation] this UDAYÁNTARA correction which is to be applied to the places of the planets, includes also the CHARA correction or the correction for the ascensional difference.

Reason of the correction called the DES'A'NTARA.

23. The places of the planets which are found being rectified by this UDAYÁNTARA correction at the time of sun-rise at LANKÁ may be found, being applied with the DESÁNTARA correction, at the time of sun-rise at a given place. This DESÁNTARA correction is two-fold, one is east and west and the other

rise in equal periods. For this reason, the Sun does not come to the horizon at LANKA' at the end of the AHARGAṆA. Therefore the places of the planets determined by the mean AHARGAṆA, will not be at the sun-rise at LANKA'. Hence a correction is necessary to be applied to the places of the planets. This correction called UDAYÁNTARA has been first invented by BHÁSHARACHÁRYA who consequently abuses them who say that the places of the planets determined by the mean AHARGAṆA become at the time of the sun-rise at LANKÁ.—B. D.]

* The difference between the mean and true AHARGAṆAS is that part of the equation of time which is due to the obliquity by the ecliptic.—L. W.

† [This calculation is nothing else than the following simple proportion
If the number of ASUS in a nycthemeron
: daily motion of the planet
: : the difference between the true and mean AHARGAṆAS
give.—B. D.]

is north and south. This north and south correction is called CHARA.

24. The line which passes from LANKÁ, UJJAYINÍ, KURUKSHETRA and other places to MERU (or the North Pole of the Earth) has been denominated the MADHYAREKHÁ mid-line of the Earth, by the Astronomers. The sun rises at any place east of this line before it rises to that line : and after it has risen on the line at places to its west. On this account, an amount of the correction which is produced from the difference between the time of sun-rise at the mid-line and that at a given place, is subtractive or additive to the places of the planets, as the given place be east or west of the mid-line [in order to find the places of the planets at the time of sun-rise at the given place].

25. As the [small] circle which is described around MERU or North Pole of the Earth, at the distance in YOJANAS reckoned from MERU to given place and produced from co-latitude of the place [as mentioned in the verse 50th, Chapter III.] is called rectified circumference of the Earth (parallel of latitude) [at that place] therefore [to find this rectified circumference], the circumference of the Earth is multiplied by the sine of co-latitude [of the given place] and divided by the radius.

End of 4th Chapter called MADHYA-GATI VASANA.

* This amount of correction is determined in the following manner.

The YOJANAS between the midline and the given place, in the parallel of latitude at that place, which is denominated SPASHTA-PARIDHI ara called, DES'A'NTARA YOJANAS of that place. Then by the proportion.

As the number of YOJANAS in the SPASHTA-PARIDHI : 60 GHATIKAS : : DESA'NTARA YOJANAS : the difference between the time of sun-rise at midline and that at a given place. This difference called DES'A'NTARA GHATIKA'S is the longitude in time east or west from LANKA'. Again

As 60 GHATIKA'S : daily motion of the planet : : DESANTA'RA GHATIKA'S : the amount of the correction required.

Or this amount can be found by using the proportion only once such as follows

As the number of YOJANAS in the SPASHTA-PARIDHI : daily motion of the planet : : DES'ÁNTARA.YOJANAS : the same amount of the correction above found.—B. D.]

CHAPTER V.

On the principles on which the Rules for finding the true places of the Planets are grounded.

On the canon of sines.

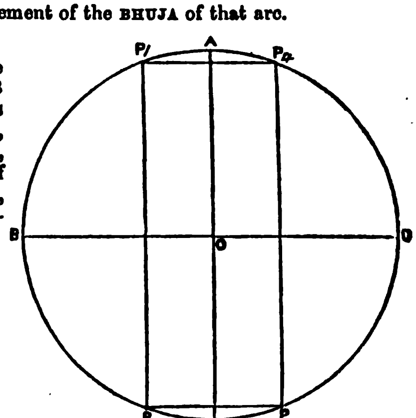
1. The planes of a Sphere are intersected by sines of BHUJA and KOṬI,* as a piece of cloth by upright and transverse threads. Before describing the spheric, I shall first explain the canon of sines.

2. Take any radius, and suppose it the hypothenuse (of a right-angled triangle). The sine of BHUJA is the base, and the sine of KOṬI is the square root of the difference of the squares of the radius and the base. The sines of degrees of BHUJA and KOṬI subtracted separately from the radius will be the versed sines of KOṬI and BHUJA (respectively).

[* The BHUJA of any given arc is that arc, less than 90°, the sine of which is equal to the sine of that given arc, (the consideration of the positiveness and negation of the sine is here neglected). For this reason, the BHUJA of that arc which terminates in the odd quadrants i. e. the 1st and 3rd is that part of the given arc which falls in the quadrant where it terminates, and the BHUJA of the arc which ends in the even quadrants, i. e. in the 2nd and 4th, is that arc which is wanted to complete the quadrant where the given arc is ended.

The KOṬI of any arc is the complement of the BHUJA of that arc.

Let the 4 quadrants of a circle A B C D be successively A B, B C, C D and D A, then the BHUJAS of the arcs A P_1, A B P_2, A C P_3, A D P_4 will be A P_1, C P_2, C P_3, A P_4 and the complements of these BHUJAS are the arcs B P_1, B P_2, D P_3, D P_4 respectively.—B. D.]



3. The versed sine is like the arrow intersecting the bow and the string, or the arc and the sine.*

The square root of half the square of the radius is the sine of an arc of 45°. The co-sine of an arc of 45° is of the same length as the sine of that arc.

* These methods are grounded upon the following principles, written by BHA'SKARA'CHARYA, in the commentary VASANA'-BHA'SHYA.

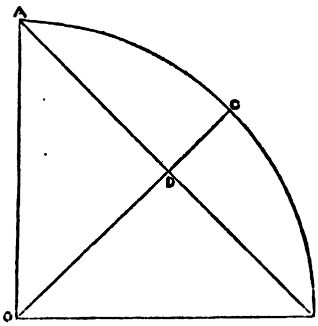
(1) Let the arc A B = 90° and A C = 45°

∴ A D (= ½ A B) = sin. 45°; and let O A or O B = the radius (R) then $AB^2 = OA^2 + OB^2 = 2\,OA^2 = 2\,R^2$

$\therefore AB = \sqrt{2R^2}$

and $AD = \frac{1}{2} AB = \sqrt{\frac{R^2}{2}}$

or $\sin. 45° = \sqrt{\frac{R^2}{2}}$.



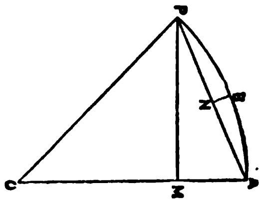
(2) It is evident and stated also in the LILA'VATI, that the side of a regular hexagon is equal to the radius of its circumscribing circle (i. e. ch. 60° = R). Hence, sin. 30° = ½ R.

(3) Let A B be the half of a given arc A P, whose sine P M and versed sine A M are given. Then

$AP = \sqrt{PM^2 + AM^2}$

and ½ A P = A N = sin. A B

$\therefore \sin. AB = \frac{1}{2}\sqrt{PM^2 + AM^2}$



(4) The proof of the last method by Algebra

cos = R — versed sine

$\therefore \cos^2 = R^2 - 2R.v + v^2$

subtracting both sides from R^2,

$R^2 - \cos^2 = 2R.v - v^2$

or $\sin.^2 = 2R.v - v^2$

adding v^2 to both sides

$\sin.^2 + v^2 = 2R.v$

and $\frac{1}{4}(\sin.^2 + v^2) = \frac{1}{2} R.v$

extracting the square root,

$$\frac{1}{2}\sqrt{\sin.^2 + v^2} = \sqrt{\frac{1}{2}R.v}$$

but by the preceding method

$\frac{1}{2}\sqrt{\sin.^2 + v^2}$ = the sine of half the given arc;

$\therefore \sin. \frac{1}{2}\text{ arc} = \sqrt{\frac{1}{2}R.v}$.—B. D.]

4. Half the radius is the sine of an arc of 30°: The co-sine of an arc of 30° is the sine of an arc of 60°.

Half the root of the sum of the squares of the sine and versed sine of an arc, is the sine of half that arc.

5. Or, the sine of half that arc is the square-root of half the product of the radius and the versed sine.

The sines and co-sines of the halves of the arcs before found may thus be found to any extent.

6. Thus a Mathematician may find (in a quadrant of a circle) 3, 6, 12, 24 &c., sines to any required extent.*

Or, in a circle described with a given radius and divided into 360°, the required sines may be found by measuring their lengths in digits.

Reason of correction which is required to find the true from the mean place of a planet.

7.† As the centre of the circle of the constellation of the Zodiac coincides with the centre of the Earth:

* [When, 24 sines are to be determined in a quadrant of a circle, the 3 sines, i. e. 12th, 8th and 16th, can be easily found by the method here given for finding the sines of 45°, 30°, and the complement of 30°, i. e. 60°. Then by means of these three sines, the rest can be found by the method for finding the sine of half an arc, as follows. From the 8th sine, the 4th and the co-sine of the 4th i. e., the 20th sine, can be determined. Again, from the 4th, the 2nd and 22nd, and from the 2nd, the 1st and 23rd, can be found. In like manner, the 10th 14th, 5th, 19th, 7th, 17th, 11th, and 13th, can also be found from the 8th sine. From the 12th again, the 6th, 18th, 3rd, 21st, 9th and 15th can be determined, and the radius is the 24th sine. Thus all the 24 sines are found. Several other methods for finding the sines will be given in the sequel.—B. D.]

[† Bha'skara'cha'rya maintains that the Earth is in the centre of the Universe, and the Sun, Moon and the five minor planets, Mars, Mercury, &c. revolve round the Earth in circular orbits, the centres of which do not coincide with that of the Earth, with uniform motion. The circle in which a planet revolves is called Prativṛitta, or excentric circle, and a circle of the same size which is supposed to have the same centre with that of the Earth, is called Kaksha'vṛitta or concentric circle. In the circle, the planet appears to revolve with unequal motion, though it revolves in the excentric with equal motion. The place where the planet revolving in the excentric appears in the concentric is its true place and to find this, astronomers apply a correction called manda-phala (1st equation of the centre) to the mean place of the planet. A mean planet thus corrected is called manda-spashṭa, the circle in which it revolves manda-pratívṛitta (1st excentric) and its farthest point from the centre of the concentric, mandochch (1st higher Apsis). As the mean places of the Sun and Moon when corrected by 1st equation become true at the centre of the Earth, this correction alone is sufficient for them. But the five minor planets, Mars, Mercury, &c. when corrected by the 1st equation are not true at the centre of the Earth but at another place. For this reason, astronomers having assumed and the centre of the circle in which the planet revolves does not coincide with the Earth : the spectator, therefore, on the Earth does not find the planet in its mean place in the Zodiac. Hence Astronomers apply the correction called BHUJA PHALA to the mean place of the planet [to get the true place].

Mode of illustration of the above fact.

8. On the northern side of a wall running due east and west, let the teacher draw a diagram illustrative of the fact for the satisfaction of his pupils.

A verse to encourage those who may be disposed to despond in consequence of the difficulties of the science.

9. But this science is of divine origin, revealing facts not cognizable by the senses. Springing from the

the concentric circle as second excentric of these five planets, take another circle of the same size and of the same centre with the Earth as concentric, and in order to find the place where the planet revolving in the 2nd excentric appears, in this concentric, they apply a correction called S'ÍGHRA-PHALA, or 2nd equation of the centre, to the mean place corrected by the 1st equation. The MANDA-SPASHTA planet, when corrected by the 2nd equation is called S'PASHTA, or true planet, the 2nd excentric, S'ÍGHRA-PRATIVRITTA, and its furthest point from the centre of the Earth, S'ÍGHROCHCH the 2nd higher Apsis.

If a man wishes to draw a diagram of the arrangement of the planets according to what we have briefly stated here, he should first describe the excentric circle, and through this excentric the concentric, and then he may determine the place of the MANDA-SPASHTA planet in the concentric thus described. Again, having assumed the concentric as 2nd excentric and described the concentric through this 2nd excentric, he may find the place of the true planet. This is the proper way of drawing the diagram, but astronomers commonly, having first described the concentric, and, through it, the excentric, find the corrected mean place of the planet in the concentric. After this, having described the 2nd excentric through the same concentric, they find the true place in the concentric, through the corrected mean place in the same. These two modes of constructing the diagram differ from each other only in the respect, that in the former, the concentric is drawn through the excentric circle, and in the latter, the excentric is drawn through the concentric, but this can easily be understood that both of these modes are equivalent and produce the same result.

In order to find the 1st and 2nd equations through a different theory, astronomers assume that the centre of a small circle called NÍCHOCHCHA-VRITTA or epicycle, revolves in the concentric circle with the mean motion of the planet and the planet revolves in the epicycle with a reverse motion equal to the mean motion. BHA'SKARA'CHA'RYA, himself will show in the sequel that the motion of the planet is the same in both these theories of excentrics and epicycles.

It is to be observed here that, in the case of the planets Mars, Jupiter and Saturn, the motion in the excentric is in fact their proper revolution, in their orbits, and the revolution of their S'ÍGHROCHCHA, or quick apogee, corresponds to a revolution of the Sun. But in the case of the planets Mercury and Venus, the revolution in the excentric is performed in the same time with the Sun, and the revolutions of their S'ÍGHROCHCHAS are in fact their proper revolutions in their orbits.—B. D.]

supreme BRAHMA himself it was brought down to the Earth by VASISHṬHA and other holy Sages in regular succession; though it was deemed of too secret a character to be divulged to men or to the vulgar. Hence, this is not to be communicated to those who revile its revelations, nor to ungrateful, evil-disposed and bad men: nor to men who take up their residence with its professors for but a short time. Those professors of this science who transgress these limitations imposed by holy Sages, will incur a loss of religious merit, and shorten their days on Earth.

Construction of a diagram to illustrate the excentric theory.

10. In the first place then, describe a circle with the compass opened to the length of the radius (3438). This is called the KAKSHÁVṚITTA, or concentric circle; at the centre of the circle draw a small sphere of the Earth with a radius equal to $\frac{1}{15}$th* of the mean daily motion of the planet.

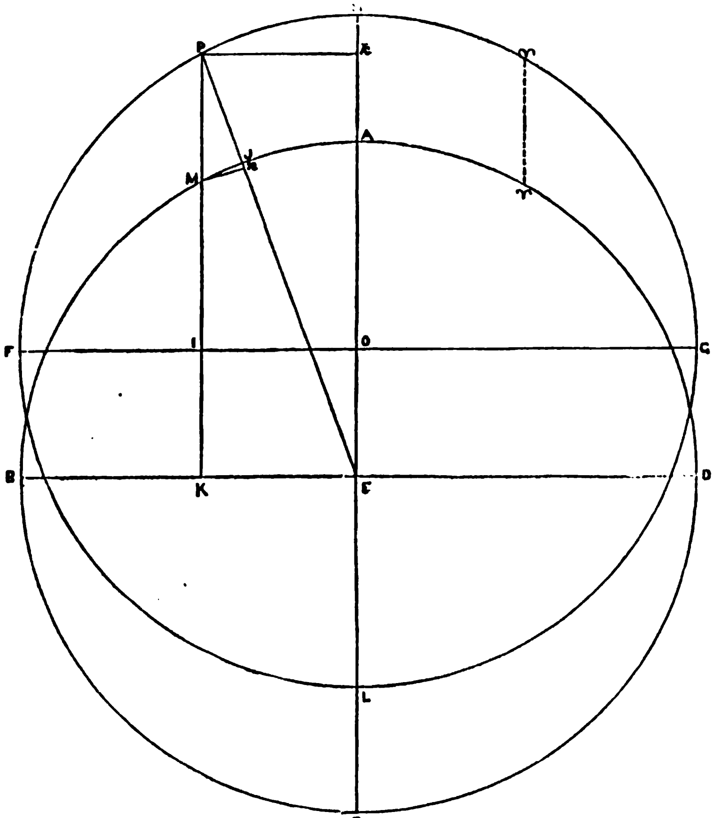
11. In this concentric circle, having marked it with 360°, find the place of the higher apsis and that of the planet, counting from the 1st point of stellar Aries; then draw a (perpendicular) diameter passing through the centre of the Earth and the higher apsis (which is called UCHCHA-REKHÁ, the line of the apsides) and draw another transverse diameter [perpendicular to the first] also passing through the centre.

12. On this line which passes to the highest apsis from the centre of the Earth, take a point at a distance from the Earth's centre equal to the excentricity or the sine of the greatest equation of the centre, and with that point as centre and the radius [equal to the radius of the concentric], describe the PRATIVṚITTA or excentric circle; the UCHCHA-REKHÁ answers the like purpose also in this circle, but make the transverse diameter different in it.

* All the Hindu Astronomers seem to coincide in thinking that the horizontal parallax PARAMA-LAMBANA of all the planets amounts to a quantity equal to $\frac{1}{15}$th of their daily motion.—L. W.

13 and 14.* Where the UCHCHA-REKHÁ perpendicular diameter (when produced) cuts the excentric circle, that is the

* Fig. 1.



[In fig. 1st let E be the centre of the concentric circle A B C D, ♈ the place of the stellar Aries, A that of the higher apsis, and M that of the mean planet in it; then E A will be the UCHCHA-REKHA (the line of the apsides). Again let E O be the excentricity and H F L G the excentric which has O for its centre; then H, ♈ P, will be the places of the higher apsis, the stellar Aries and the planet respectively in it. Hence H P will be the KENDRA; P K the sine of the KENDRA; P I the co-sine of the KENDRA.

The KENDRA which is more than 9 signs and less than 3 is called MRIGADI (i. e. that which terminates in the six signs beginning with Capricornus) and that which is above 3 and less than 9 is called KARKYADI (i. e. that which ends in the six signs beginning with Cancer).

Thus (Fig. 1) that which terminates in G H F is MRIGADI KENDRA, and that which ends in F L G is KARKYA'DI.—B. D.]

place of the higher apsis in it also. From this mark the first stellar Aries, at the distance in degree of the higher apsis in antecedentia: the place of the planet must be then fixed counting the degree from the mark of the 1st Aries in the usual order.

The distance between the higher apsis and the planet is called the KENDRA.* The right line let fall from the planet on the UCHCHA-REKHÁ is the sine of BHUJA of the KENDRA. The right line falling from the planet on the transverse diameter is the cosine of the KENDRA, it is upright and the sine of BHUJA is a transverse line.

The principle on which the rule for finding the amount of equation of centre is based.

15. As the distance between the diameters of the two circles is equal to the excentricity and the co-sine of the KENDRA is above and below the excentricity when the KENDRA is MRIGÁDI and KARKYÁDI (respectively).†

* The word KENDRA or centre is evidently derived from the Greek word κεντρον and means the true centre of the planet.—L. W.

† [In (Fig. 1) P K is the SPHUṬA KOṬI and P E the KARṆA (the hypothenuse) which cuts the concentric at T. Hence the point T will be the apparent place of the planet and T M the equation of the centre.

This equation can be determined as follows.

Draw M n perpendicular to E T, it will be the sine of the equation and the triangle P M n will be similar to the triangle P E K.

$$\therefore \text{P E} : \text{E K} = \text{P M} : \text{M}\,n\,;$$

$$\text{hence M}\,n = \frac{\text{P M . E K}}{\text{P E}} = \text{sine of the equation ;}$$

$$= \frac{\text{E O . E K}}{\text{P E}}, \text{ for P M} = \text{I K} = \text{E O}$$

Now, let k = KENDRA, a = the distance between the centres of the two circles excentric and concentric, x = sine of the equation, and h = hypothenuse: then the SPHUTA KOṬI = cos. $k \pm a$, according as the KENDRA is MRIGADI or KARKYADI, and $h = \sqrt{\sin.^2 k \pm (\cos. k \pm a)^2}$
hence by substitution

$$x = \frac{a \,.\, \sin. k}{h} = \frac{a \,.\, \sin. k}{\sqrt{\sin.^2 k + (\cos. k \pm a)^2}}$$

16 and 17. Therefore the sum or difference of the co-sine and excentricity (respectively) is here the SPHUṬA KOṬI (i. e. the upright side of a right-angled triangle from the place of the planet in the excentric to the transverse diameter in the concentric,) the sine of the BHUJA [of the KENDRA] is the BHUJA (the base) and the square-root of the sum of the squares of the SPHUṬA KOṬI and BHUJA is called KARṆA, hypothenuse. This hypothenuse is the distance between the Earth's centre and the planet's place in the excentric circle.

The planet will be observed in that point of the concentric cut by the hypothenuse.

The equation of the centre is the distance between the mean and apparent places of the planet: when the mean place is more advanced than the apparent place then the equation thus found is subtractive; when it is behind the true place, the equation is additive.*

The reason for assuming the MAND-SPASHṬA planet as a mean in finding the 2nd equation.

18. The mean planet moves in its MANDA-PRATIVṚITTA (first excentric); the MANDA-SPASHṬA planet (i. e. whose mean place is rectified by the first equation) moves in its S'IGHRA-PRATIVṚITTA (second excentric). The MANDA-SPASHṬA

It also follows from this that, when cos. k is equal to a in the KARKYADI KENDRA, then h will be equal to sin. k, otherwise h will always be greater than sin. k and consequently x will be less than a. Hence, when h is equal to sin k, x will then be greatest and equal to a, i. e. the equation of the centre will be greatest when the hypothenuse is equal to the sine of the KENDRA, or when the planet reaches the point in the excentric cut by the transverse line in the concentric. Therefore, the centre of the excentric is marked at the distance equal to the excentricity from the centre of the concentric (as stated in the V 12th.)—B. D.]

* [Thus, the mean planet, corrected by the 1st equation, becomes MANDA-SPASHṬA and this process is called the MANDA process. After this, the MANDA-SPASHṬA when rectified by the SI'GHRA PHALA, or 2nd equation, is the SPASHṬA planet, and this 2nd process is termed the S'IGHRA process. Both of these processes, MANDA-SPASHṬA and SPASHṬA are reckoned in the VIMAN-ḌALA or the orbit of the planet as hinted at by BHASKARACHARYA in the commentary called VASANA-BHASHYA in the sequel. These places are assumed for the ecliptic also without applying any correction to them, because the correction required is very small.—B. D.]

is therefore here assumed as the mean planet in the second process (i. e. in finding the second equation).*

The reason for the invention of the higher apsis.

19. The place in the concentric in which the revolving planet in its own excentric is seen by observers is its true place. To find the distance between the true and mean places of the planet, the higher apsis has been inserted by former Astronomers.

20. That point of the excentric which is most distant from the Earth has been denominated the higher apsis (or UCHCHA): that point is not fixed but moves; a motion of the higher apsis has therefore been established by those conversant with the science.

21. The lower apsis is at a distance of six signs from the higher apsis: when the planet is in either its higher or lower apsis, then its true place coincides with its mean place, because the line of the hypothenuse falls on the mean place of the planet in the concentric.

The cause of variation of apparent size of planet's disc.

22. As the planet when in the higher apsis is at its greatest distance from the Earth, and when in the lower apsis at its least distance, therefore its disc appears small and large accordingly. In like manner, its disc appears small and large accordingly as the planet is near to and remote from the Sun.

23. To prevent the student from becoming confused, I have separately explained the proof of finding the equation by the PRATIVRITTA BHANGI of the diagram of the excentric. I shall now proceed to explain the same proof in a different manner by the diagram of a NÍCHOCHCHA-VRITTA (epicycle).

* [For this reason, having assumed the MANDA-SPASHṬA planet for the mean, which MANDA-SPASHṬA can be determined in the concentric by describing the excentric circle &c. through the mean planet and MANDOCHCHA, make the place of the stellar Aries from the MANDA-SPASHṬA place in the reverse order of the signs and then determine the place of the S'ÍGHROCHCHA in the order of the signs. Through the places of the stellar Aries and S'ÍGHROCHCHA describe the 2nd excentric circle &c. in the way mentioned before, and then find the place of the true planet in the concentric.—B. D.]

Construction of Diagram to illustrate the theory of epicycle.

24. Taking the mean place of the planet in the concentric as the centre, with a radius equal to the excentricity of the planet, draw a circle. This is called NÍCHOCHCHA VRITTA or epicycle. Then draw a line from the centre of the Earth passing through the mean place of the planet [to the circumference of the epicycle].

25. That place in the epicycle most distant from the centre of the Earth, cut by the line [joining the centre of the Earth and mean place of the planet] is supposed to be the place of the higher apsis: and the point in the epycicle nearest to the Earth's centre, the lower apsis. In the epicycle draw a transverse line passing through the centre of it [and at right-angles to the above-mentioned line which is called here UCHCHA-REKHÁ].

26. As the mean planet revolves with its KENDRA-GATI (the motion from its higher apsis) in the 1st and 2nd epicycle marked with the 12 signs and 360 degrees towards the reverse signs, and according to the order of the signs respectively from its higher apsis.

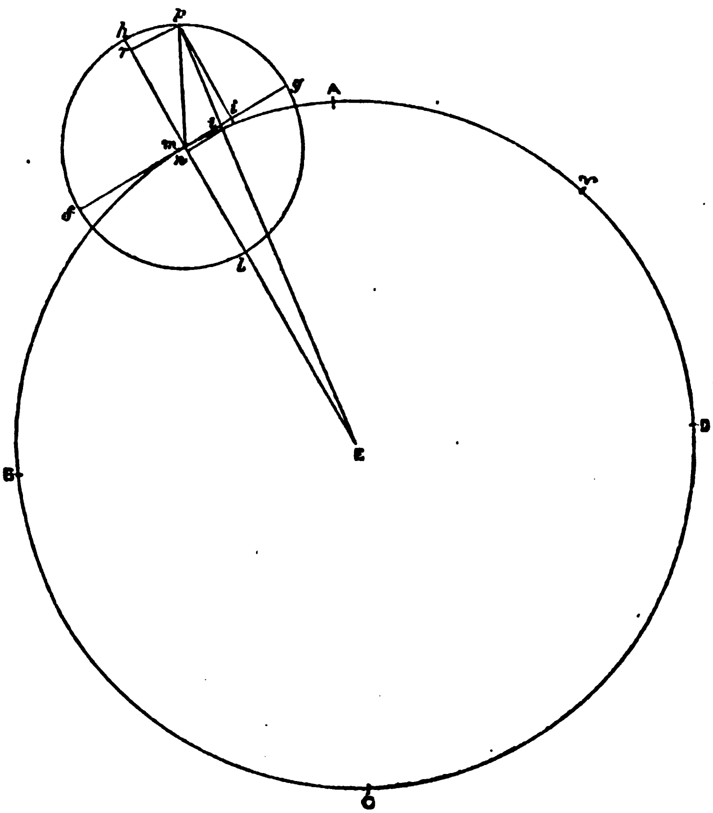
27. Mark off therefore the places of the first and second KENDRAS or distances from their respective higher apsides in the manner directed in the last verse: the planet must be fixed at those points. [Here also] The (perpendicular) line from the planet to the UCHCHA-REKHÁ is the sine of the BHUJA of the KENDRA: and from the planet on the transverse line is the cosine [of the KENDRA].* *(See note next page.)*

To find the hypothenuse and the equation of centre.

28 and 29. The BHUJA PHALA and KOṬI PHALA of the KENDRA which are found [in the GANITÁDHYÁYA] are sine and cosine in the epicycle. As the KOṬI PHALA is above the radius (of the concentric) in MRIGÁDI KENDRA and within the radius in KARKYÁDI-KENDRA, the sum and difference, therefore, of the KOṬI PHALA and the radius is here the SPHUTA-KOṬI (upright line), the BHUJA PHALA is the BHUJA (the base) and the KARṆA hypothenuse (to complete

the right-angled triangle) is the line intercepted between the centre of the Earth and the planet. The equation of the centre is here the arc [of the concentric] intercepted between

* Note on verses from 24 to 27.



[In fig. 2, let A B C D be the concentric, ♈ the place of the stellar Aries, E the centre of the Earth, M the mean place of the planet in the concentric, *h f l g*, the Epicycle, *h* the place of the higher apsis in it, E *h* the UCHCHA-REKHA' *l* the place of the lower apsis, P that of the planet, *h* P the KENDRA, P *k* the sine of the KENDRA and P *i* the cosine of it.

The sine and co-sine of the KENDRA in the excentric, reduced to their dimensions in the epicycle in parts of the radius of the concentric, are named BHUJA-PHALA and KOṬI-PHALA respectively in the GAṆITÁDHYÁYA. That is

As the radius or 360° of the concentric
: the sine and cosine of the KENDRA in the excentric
: : excentricity or the periphery of the epicycle
: BHUJA-PHALA and KOṬI-PHALA respectively.

Therefore the BHUJA-PHALA and KOṬI-PHALA must be equal to the sine and cosine of the KENDRA in the epicycle.—B. D] the mean place of the planet and the point cut by the hypothenuse. The equation thus found is to be added or subtracted as was before explained.*

Construction of the mixed diagrams of the excentric and epicycle.

30. The planet appears to move forward from MANDOCHCHA, or 1st higher apsis, in the excentric circle with its KENDRA-GAṬI (the motion from its MANDOCHCHA) and in the order of the signs and to the East: From its SI'GHROCHCHA, 2nd higher apsis, it moves in antecedentia or reversely, as it is thrown backwards.

31. When the epicycle however is used, the reverse of this takes place, the planet moving in antecedentia from its 1st higher apsis and in the order of the signs from its 2nd higher apsis. Now as the actual motion in both cases is the same, while the appearances are thus diametrically opposed, it must be admitted therefore that these expedients are the mere inventions of wise astronomers to ascertain the amount of equation.

* In (Fig 2) E k is the SPHUṬA-KOṬI, P E the hypothenuse, T the apparent place of the planet in the concentric and T M the equation of the centre. This equation can also be found by the theory of the epicycle in the following manner.

Draw T s perpendicular to E M, then T s will be the sine of the equation; let it be denoted by x, the KENDRA in the excentric by k, the excentricity by a, and the hypothenuse by h: then

$$\text{R} : \sin k = a : \text{P}\,k \text{ the BHUJA-PHALA}$$

$$\therefore \text{ the BHUJA-PHALA} = \frac{a \sin k}{\text{R}},$$

Now, the triangles E T s and E P k are similar to each other

$$\therefore \text{E P} : \text{P}\,k = \text{E T} : \text{T}\,s$$
$$\text{or } h : \text{P}\,k = \text{R} : x$$
$$\therefore x = \frac{\text{P}\,k \times \text{R}}{h};$$

that is, the BHUJA-PHALA multiplied by the radius and divided by the hypothenuse is equal to the sine of the equation.

$$\text{But P}\,k = \frac{a \sin k}{\text{R}};$$

$\therefore$ by substitution

$$x = \frac{a \sin k}{\text{R}} \times \frac{\text{R}}{h} = \frac{a \sin k}{h},$$ the sine of the equation as found before by the theory of the excentric in the note on the verses 15, 16 and 17.—B. D.]

32. If the diagrams (of the excentric and epicycle) be drawn unitedly, and the place of the planet be marked off in the manner before explained, then the planet will necessarily be in the point of the intersection of the excentric by the epicycle.

33. [In illustration of these opposite motions, examine an oil-man's screw-press.] As in the oil-man's press, the wooden press (moving in the direction in which the bullock fastened to it goes) moves (also itself) in the opposite direction to that in which the bullock goes, thus the motion of the planet, though it moves in the excentric circle, appears in antecedentia in the epicycle.

34. As the centre of the 1st epicycle is in the concentric,

Explains why the 5 minor planets require both the 1st and 2nd equations to their true places.

let the planet therefore move in the concentric with its mean motion: In the concentric [at that point cut by the first hypothenuse] is the centre of the SI'GHRA NÍCHOCHCHA, VṚITTA or of the 2nd epicycle: In the second or S'IGHRA epicycle is found the true place of the planet.

35. The first process, or process of finding the 1st equation, is used in the first place, in order to ascertain the position of the centre of the SÍGHRA NÍCHOCHCHA VṚITTA or of the 2nd epicycle, and the 2nd process, or the process of the 2nd equation, to ascertain the actual place of the planet. As these two processes are mutually dependent, it on this account becomes necessary to have recourse to the repetition of these two processes.

36 and 37. Some say that the hypothenuse is not used in

Explains reason of omission of hypothenuse in the MANDA process.

the 1st process, because the difference (in the two modes of computation) is inconsiderable, but others maintain that since in this process the periphery of the first epicycle being multiplied by the hypothenuse and divided by the radius becomes true, and that, if the hypothenuse then be used, the result is the same as it was before, therefore the hypothenuse is not employed. No objection is to be made why this is not the case in the 2nd process, because the proofs of finding the equation are different here.*

38. As no observer on the surface of the Earth sees the planet moving in the excentric, deflected from his zenith, in that place of the concentric, where an observer situated at the centre of the Earth observes it in the eastern or western hemisphere, and at noon both observers see it in the same place, therefore the correction called NATAKARMA is declared (by astronomers). The proof of this is the same as in finding the parallax.†

Reason of NATAKARMA.

* [The BHUJA-PHALA, determined by means of the sine of the first KENDRA of the planet (i. e. by multiplying it by the periphery of the 1st epicycle and dividing it by 360°) has been taken for the sine of the 1st equation of the centre: and what we have shown in the note on the V. 28 and 29, that the BHUJA-PHALA, when multiplied by the radius and divided by the hypothenuse, becomes the sine of the equation may be understood only for finding the 2nd equation of the five minor planets and not for determining the 1st equation.

Some say that the omission of the hypothenuse in the 1st process has no other ground but the very inconsiderable difference of the result. But BRAHMAGUPTA maintains that the periphery of the 1st epicycle, varies according to the hypothenuse; that is, their ratio is always the same, and the periphery of the 1st epicycle, mentioned in the GAṆITÁDHYÁYA, is found at the instant when the hypothenuse is equal to the radius. For this reason, it is necessary at first to find the true periphery through the hypothenuse and then determine the 1st equation. But, he declares that by so doing; also the sine of the equation becomes equal to the BHUJA-PHALA as follows.

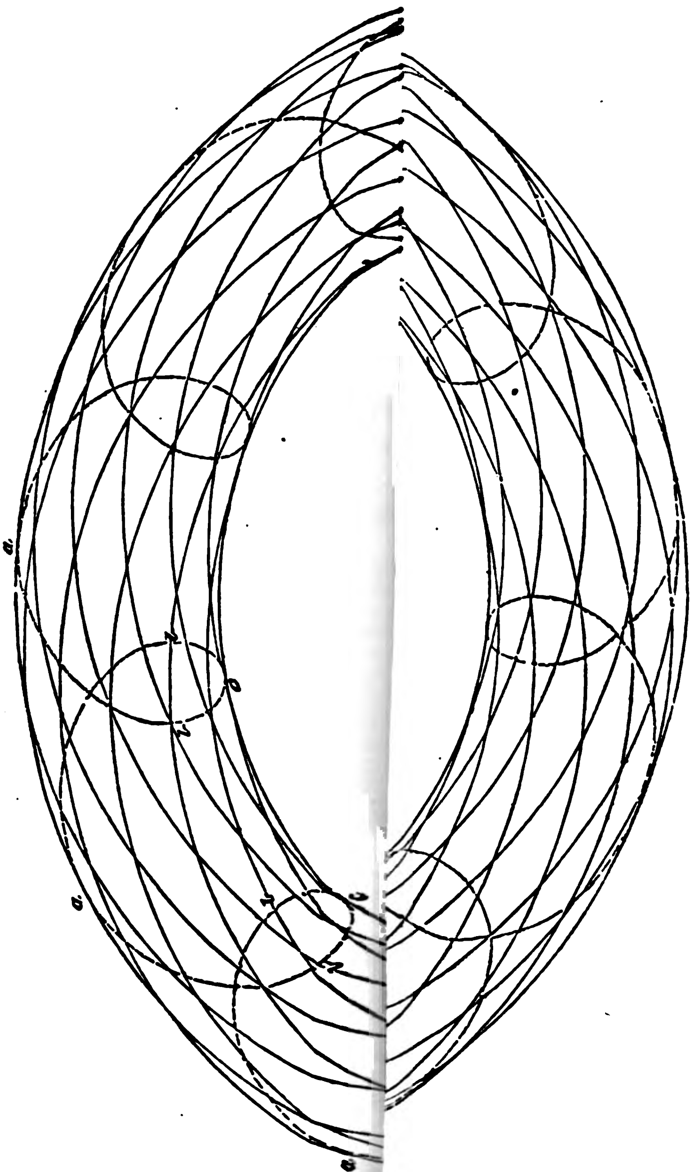
As R : 1st periphery = the hypothenuse : the true periphery

$\therefore$ the true periphery $= \dfrac{P \times h}{R}$, and consequently the BHUJA-PHALA in the true epicycle $= \dfrac{P \times h}{R} \times \dfrac{\sin k}{360^\circ}$;

$\therefore$ the sine of the 1st equation $= \dfrac{P \times h}{R} \times \dfrac{\sin k}{360^\circ} \times \dfrac{R}{h}$ and abridging $= \dfrac{P \,.\, \sin k}{360^\circ}$ which is equal to the BHUJA-PHALA. Hence the hypothenuse is not used in the 1st process.

BRAHMAGUPTA'S opinion is much approved of by BHA'SKARA'CHA'RYA.—B. D.]

† But this is not the case, because the NATAKARM which BHA'SKARA'CHA'RYA has stated in the GAṆITÁDHYÁYA has no connection with the fact stated in this S'LOKA and therefore many say that this S'LOKA does not belong to the text.—B. D.]



39. The mean motion of a planet is also its true motion when the planet reaches that point in the excentric cut by the transverse diameter which passes through the centre of the concentric: and it is when the planet is at that point that the amount of equation is at its maximum. [LALLA has erroneously asserted that the mean and true motions coincide at the point where the concentric is cut by its excentric.]*

Explains where the mean and true motions of all the planets coincide.

40. Having made the excentric and other circles of thin pieces of bamboo in the manner explained before, and having changed the marks of the places of the planet and its S'ÍGHROCHCHA 2nd higher apsis with their daily motions, an astronomer may quickly show the retrogressions, &c.†

Manner of observing the retrogression &c. of Planets.

* The ancient astonomers LALLA, S'RIPATI &c. say that the true motion of a planet equals to its mean motion when it reaches the point of intersection of the concentric and excentric. But BHA'SKARA'CHARYA denying this, says, that when the planet reaches the point when the transverse axis of the concentric cuts the excentric and when the amount of equation is a maximum, the true motion of a planet becomes equal to its mean motion. For, suppose, p_1, p_2, p_3, &c., are the mean places of a planet found on successive days at sun-rise when the planet proceeded from its higher or lower apsis and e_1, e_2, e_3, &c. are the amounts of equation, then $p_1 \pm e_1$, $p_2 \pm e_2$, $p_3 \pm e_3$, &c. will be the true places of the planet,
$\therefore p_2 - p_1 \pm (e_2 - e_1), p_3 - p_2 \pm (e_3 - e_2), p_4 - p_3 \pm (e_4 - e_3)$, &c. will be the true motions of the planet on successive days. Now, as the difference between the true and mean motions is called the GATIPHALA, by cancelling therefore, $p_2 - p_1$, $p_3 - p_2$, &c. the parts of the true motions which are equal to the mean motion, the remaining parts $e_2 - e_1$, $e_3 - e_2$ &c. will evidently be the GATIPHALAS that is the differences between two successive amounts of equation are the GATIPHALAS. Thus, it is plain that the GATIPHALA entirely depends upon the amount of equation, but as the amount of equation increases, so the GATIPHALA is decreased and therefore when it is a maximum, the GATIPHALA will indifinitely be decreased i. e. will be equal to nothing. Now as the amount of equation becomes a maximum in that place where the transverse diameter of the concentric circle cuts the excentric, (see the note on verses 15, 16 and 17) the GATIPHALA, therefore becomes equal to nothing at the same place, that is, in that very place, the true motion and mean motions of a planet are equal to each other. Having thus shown a proof of his own assertion, BHÁSKARA'CHA'RYA says that what the ancient astronomers stated, that the true and mean motions of a planet are equal to each other when the planet comes in the intersection point of the concentric and excentric circles, is entirely ungrounded.—B. D.]

† According to the method above mentioned, if the place of the higher apsis and that of the planet be changed, and the planet's place be marked, the motion of the planet will be in a path like the dotted line as shown in the diagram.

See Diagram facing this page.

41. The word KENDRA (or *κεντρον*) means the centre of a circle: it is on that account applied to the distance between the planet and higher apsis, for the centre of the NICHOCHCHA-VṚITTA or epicycle, is always at the distance of the planet from the place of the higher apsis.

The reason of the invention of the appellation of KENDRA.

42. The circumference in YOJANAS of the planet's orbit being multiplied by the S'ÍGHRA-KARṆA (or 2nd hypothenuse), and divided by the radius (3438) is SPHUṬA-KAKSHÁ (corrected orbit). The planet is (that moment) being carried [round the earth] by the PRAVAHA wind, and moves at a distance equal to half the diameter of the SPHUTA-KAKSHÁ from the earth's centre.

SPHUṬA-KAKSHA or corrected orbit.

43. When the sun's MANDA-PHALA i. e. the equation of the centre is subtractive, the apparent or real time of sun-rise takes place before the time of mean sun-rise: when the equation of the centre is additive, the real is after the mean sun-rise, on that account the amount of that correction arising from the sun's MANDA-PHALA converted into ASUS* of time has been properly declared to be subtractive or additive.

Reason of BUJA'NTARA correction.

44. Those who have wits as sharp as the sharp point of the inmost blade of the DORBHA or DARBHA grass, find the subject above explained by diagrams, a matter of no difficulty whatever: but men of weak and blunt understanding find this subject as heavy and immovable as the high mountain† that has been shorn of its wings by the thunderbolt of Indra.

End of Chapter V. on the principles on which the rules for finding the true places of the planets are grounded.

It is to be observed here that when the planet comes to the places *a*, *a* &c. in the dotted line, it is then at its higher apsis, when it comes to the places *c*, *c* and *c*, it is at its lower, and when it comes to *b*, *b* &c. it appears, stationary: and when it is moving in the upper arc *b a b*, its motion being direct appears quicker, and when in the lower arc *b c b*, its retrograde motion is seen.—B. D.]

* [These ASUS are equivalent to that part of the equation of time, which is due to the unequal motion of the sun on the ecliptic.—B. D.]

† Mountains are said by Hindu theologians to have originally had wings.

CHAPTER VI.

Called GOLABANDHA, *on the construction of an Armillary Sphere.*

1. Let a mathematician, who is as skilful in mechanics as in his knowledge of the sphere, construct an armillary sphere with circles made of polished pieces of straight bamboo; and marked with the number of degrees in the circle.

2. In the first place, let him mark a straight and cylindrical DHRUVA-YASHTI, or polar axis, of any excellent wood he pleases: then let him place loosely in the middle of it a small sphere to represent the earth [so that the axis may move freely through it]. Let him then firmly secure the spheres beyond it of the Moon, Mercury, Venus, the Sun, Mars, Jupiter, Saturn and the fixed stars: Beyond them let him place two spheres called KHAGOLA and DṚIGGOLA unconnected with each other, and fastened to the hollow cylinders [in which the axis is to be inserted].*

[Description in detail of the fact above alluded to.]

The prime vertical, the meridian and the KOṆAVRITTAS.

3. Fix vertically the four circles and another circle called horizon transversely in the middle of them, so that one of those vertical circles called SAMAMAṆḌALA, prime vertical, may pass through the east and west points of the horizon, the other called YÁMYOTTARA-VRÍTTA, meridian the

* The sphere of the fixed stars which is mentioned here is called the BHAGOLA starry sphere. This BHAGOLA is assumed for all the planets, instead of fixing a separate sphere for each planet. This sphere consists of the circles ecliptic, equinoctial, diurnal circles, &c. which are moveable. For this reason, this sphere is to be firmly fixed to the polar axis, so that it may move freely by moving the axis. Beyond this sphere, the KHAGOLA celestial sphere which consists of the prime vertical, meridian, horizon, &c. which remain fixed in a given latitude is to be attached to the hollow cylinders. Having thus separately fixed these two spheres, astronomers attach, beyond these, a third sphere in which the circles forming both the spheres KHAGOLA and BHAGOLA are mixed together. For this reason the latter is called DṚIGGOLA the double sphere. And as the spherical fingers are well seen by mixing together the two spheres KHAGOLA and BHAGOLA, the third sphere which is the mixture of the two spheres, is separately attached.—B. D.]

north and south points, and the remaining two called KOṆA-VṚITTAS the N. E. and S. W. and N. W. and S. E. points.

The UNMAṆḌALA or six o'clock line.

4. Then fix a circle passing through the points of the horizon intersected by the prime vertical, and passing also through the south and north poles at a distance below and above the horizon equal to the latitude of the place. This is called the UNMAṆḌALA, or six o'clock line, and is necessary to illustrate the increase and decrease in the length of the days and nights.*

The equinoctial.

5. The equinoctial (called NÁḌÍ-VALAYA), marked with 60 ghaṭis, should be placed so as to pass through the east and west points of the horizon, and also to pass over the meridian at a distance south from the zenith equal to the latitude, and at a distance north of the nadir also equal to the latitude of the place [for which the sphere is constructed].

Azimuth or vertical circle.

6. Let the azimuth or vertical circle be next attached within the other circles, fixed by a pair of nails at the zenith and nadir, so as to revolve freely on them: [It should be smaller than the other circles so as to revolve within them]. It should be capable of being placed so as to cover the planet, wherever it may happen to be.

7. Only one azimuth circle may be used for all the planets; or else eight azimuth circles may be made, viz. one for each of the 7 planets and the 8th for the nonagesimal point. The azimuth circle for the nonagesimal point is called the DṚIK-SHEPA-VṚITTA.

* The circle of declination or the hour circle passing through the east and west points of the horizon is called UNMAṆḌALA in Sanskrit; but I am not acquainted with any corresponding term in English. In the treatise on astronomy in the Encyclopædia Metropolitana the prime vertical is named the six o'clock line. This term (six o'clock line) should, I think, be applied to the UNMAṆḌALA, because it is always six o'clock when the sun arrives at this circle, the UNMAṆḌALA. The prime vertical or the SAMA-MAṆḌALA of the Sanskrit cannot, with propriety, be called the six o'clock line: because it is only twice a year that it is six o'clock when the sun is at this circle, the prime vertical.—B. D.]

8. Let two hollow cylinders project beyond the two poles north and south of the KHAGOLA celestial sphere, and on those cylinders let the skilful astronomer place the DṚIGGOLA double sphere as follows.

The DṚIGGOLA.

9. When the system of the KHAGOLA, celestial sphere, is mixed with the ecliptic, and all the other circles forming the BHAGOLA (which will be presently shown) it is then called DṚIGGOLA, double sphere. As in this the figures formed by the circles of the two spheres KHAGOLA and BHAGOLA are seen, it is therefore called DṚIGGOLA double sphere.*

THE BHAGOLA.

10. Let two circles be firmly fixed on the axis of the poles answering to the meridian and horizon (of the KHAGOLA); they are called the ÁDHÁRA-VṚITTAS, or circles of support: Let the equinoctial circle also be fixed on them marked with 60 ghaṭis like the prime vertical (of the KHAGOLA).

11. Make the ecliptic (of the same size) and mark it with 12 signs; in this the Sun moves: and also in it revolves the Earth's shadow at a distance of 6 signs from the Sun. The KRÁNTI-PÁTA or vernal equinox, moves in it contrary to the order of the signs: The SPASHṬA-PÁTAS [of the other planets] have a like motion: the places of these should be marked in it.†

The Ecliptic.

* See the note on 2 Verse.

† [The Sun revolves in the ecliptic, but the planets, Moon, Mars, &c. do not revolve in that circle, and the planes of their orbits are inclined to that of the ecliptic. Of the two points where the planetary orbit cuts the plane of the ecliptic, that in which the planet in its revolution rises to the north of the ecliptic is called its PA'TA or ascending node (it is usually called the mean PA'TA) and that which is at the distance of six signs from the former is called its SASHADBHA PA'TA or descending node. The PA'TA of the Moon lies in its concentric, because the plane of its orbit passes through the centre of the concentric, i. e. through the centre of the Earth; but the PA'TAS of the other planets are in their second excentric, because the planes of their orbits pass through the centres of their 2nd excentrics, which centres lie in the plane of the ecliptic. When the planet is at any other place than its nodes, the distance between it and the plane of the ecliptic is called its north or south latitude as the planet is north or south of the ecliptic. When the planet is at the distance of 3 signs forward or backward from its PA'TA, it is then at the greatest distance north or south from the ecliptic: This distance is its greatest latitude. Thus,

12. Let the ecliptic be fixed on the equinoctial in the point of vernal equinox KRÁNṬI-PÁTA and in a point (autumnal equinox) 6 signs from that: it should be so placed that the point of it, distant 3 signs eastward from the vernal equinox, shall be 24° north of the equinoctial, and the 3 signs westward shall be at the same distance south from the equinoctial.

Planet's orbit.

13. Divide a circle called KSHEPA-VṚITTA representing the orbit of a planet into 12 signs and mark in it the places of the SPASHTA-PÁTAS, rectified nodes, as has been before prescribed [for the ecliptic]. Then this circle should be so placed in connection with the ecliptic as it has been placed in connection with the equinoctial.

14. The ecliptic and the KSHEPA-VṚITTA should be so placed that the latter may intersect the former at the [rectified] ascending and descending nodes, and pass through points distant 3 signs from the ascending node east and west at a distance from the ecliptic north and south equal to the rectified greatest latitude of the planet [for the time].

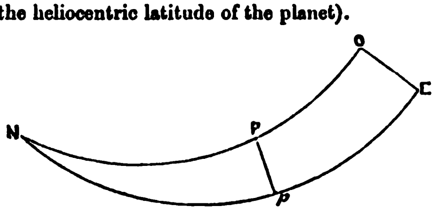
15. The greatest (mean) latitudes of the planets being multiplied by the radius and divided by the SÍGHRA-KARṆA

the latitude of the planet begins from its PA'TA and becomes extreme at the distance of 3 signs from it, therefore, in order to find the latitude, it is necessary to know the distance between the planet and its PA'TA. This distance is equal to the sum of the places of the planet and its PA'TA, because all PA'TAS move in antecedentia from the stellar aries. This sum is called the VIKSHEPA-KENDRA or the argument of latitude of the planet. As the PA'TA of the Moon lies in her concentric, and in this circle is her true place, the sum of these two is her VIKSHEPA-KENDRA, but the PA'TA of any other planet, Mars, &c. lies in its 2nd excentric and its MANDA-SPASHTA place (which is equivalent to its heliocentric place) is in that circle, therefore its VIKSHEPA-KENDRA is found by adding the place of its PA'TA to its MANDA-SPASHTA place. The SPASHTA-PA'TA of the planet is that which being added to the true place of the planet, equals its VIKSHEPA-KENDRA for this reason, it is found by reversely applying the 2nd equation to its mean PA'TA. As

∴ SPASHTA PA'TA + true place of the planet,
= VIKSHEPA-KENDRA,
= place of the MANDA SPASHTA planet + mean PA'TA,
= p, of the $m. s. p.$ $\pm$ 2nd equation + $m.$ P $\mp$ 2nd equation,
= true place of the planet + mean PA'TA $\pm$ 2nd equation,
∴ SPASHTA PA'TA = mean PA'TA $\mp$ 2nd equation.

The place of this SPASHTA PA'TA is to be reversely marked in the ecliptic from the stellar aries. — B. D] second hypothenuse becomes SPASHṬA, rectified. The KSHEPA-VṚITTA, or circles representing the orbits of the six planets, should be made separately. The Moon and the rest revolve in their own orbits.*

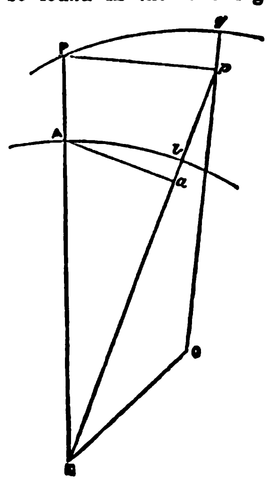
* [As the PA'TA of the Moon and her true place lie in her concentric, the sum of these two, which is called her VIKSHEPA-KENDRA or the argument of latitude, must be measured in the same circle, and her latitude, therefore found through her VIKSHEPA-KENDRA, will be as seen from the centre of her concentric i. e. from the centre of the Earth. But the PA'TA of any other planet and its MANDA-SPASHTA place (which is its heliocentric place) lie in its 2nd excentric, therefore its latitude, determined by means of its VIKSHEPA-KENDRA, which is equal to the sum of its MANDA-SPASHTA place and PA'TA and measured in the same circle, will be such as seen from the centre of its 2nd excentric and is called its mean latitude (which is equivalent to the heliocentric latitude of the planet).



As in Fig. 1, let N E be the quarter of the ecliptic, N O that of the 2nd excentric, N the node and P the planet. Suppose O E and P p (parts of great circles) to be drawn from O and P perpendicularly to the plane of the ecliptic: then O E will be the greatest latitude and P p the latitude of the planet at P, by which a spectator at the centre of the 2nd excentric and not at the centre of the Earth, will see the planet distant from the ecliptic. This latitude, therefore, is called a mean latitude which can be found as follows,

$$\sin \text{N O} : \sin \text{O E} :: \sin \text{N P} : \sin \text{P}p,$$
$$\text{or R} . \sin \text{P}p = \sin \text{O E} . \sin \text{N}p,$$

consequently, in order to determine P p, it is necessary to know previously O E, the greatest latitude and N P, the distance of the place of the planet from the node, which distance is evidently equal to the VIKSHEPA-KENDRA that is, to the sum of the MANDA-SPASHTA place of the planet and the mean place of the node. Now the latitude of the planet as seen from the centre of the Earth is called its true latitude. This true latitude can be found in the following manner,



Let E be the centre of the earth, O that of the 2nd excentric, P the MANDA SPASHTA place of the planet in it: then E P will be the 2nd hypothenuse which is supposed to cut the concentric at A: then A will be the true place of the planet in the concentric. Again let P q be a circle with the centre O, whose plane is perpendicular to the ecliptic plane and A b another circle with the centre E whose place is also perpendicular to the same plane, then P q will be the mean latitude of the planet and A b will be the true. Let P p and A a lines be perpendicularly drawn to the plane of the ecliptic, these lines will also be at right angles to the line E p: then P p will be the sine of the mean latitude P q and A a that of the true latitude A b. Now by the similar triangles E P p and E A a,

$$\text{E P} : \text{P}p :: \text{E A} : \text{A}a;$$
$$\therefore \text{A}a = \frac{\text{E A} . \text{P}p}{\text{E P}};$$

16. The declination is an arc of a great meridian circle: cutting the equinoctial at right angles, and continued till it touch the ecliptic.

Declination and latitude.

or the sine of the true latitude $= \dfrac{R \times \text{sine of the mean latitude}}{h}$

but, the sine of the mean latitude $= \dfrac{\sin OE \,.\, \sin NP}{R}$

$\therefore$ by substitution

the sine of the true latitude $= \dfrac{R}{h} \times \dfrac{\sin OE \,.\, \sin NP}{R}$

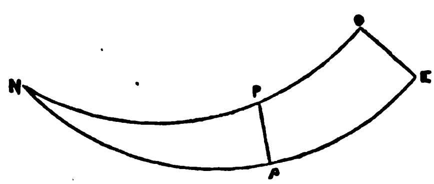
$$= \frac{\sin OE \,.\, \sin NP}{h}$$

As the latitude of the planet is of a smaller amount, the arc of a latitude it, therefore taken in the SIDDHANTAS instead of the sine of the latitude.

Hence, the true latitude $= \dfrac{OE \,.\, \sin NP}{h}$,

that is, the sine of the argument of latitude multiplied by the greatest latitude and divided by the 2nd hypothenuse is equal to the true latitude of the planet.

Now in the BHAGOLA, a circle should be so fixed to the ecliptic, that the former may intersect the latter at the SPASUTA-PÁTA and the point six signs from it, and whose extreme north and south distance from the ecliptic may be such that the distance between the circle and the ecliptic at the place of the true planet may be equal to the true latitude of the planet. This circle is called the VIMANḌALA or VIKSHEPA-VṚITTA and its extreme north and south distance from the ecliptic is called the true or rectified extreme latitude of the planet which can be found as follows.



Let N be the SPASHTA-PATA, N P the VIKSHEPA-KENDRA, P p the true latitude, E O the true extreme latitude: then

$\sin N o : \sin E O :: \sin NP : \sin Pp$

$$\therefore \sin EO = \frac{\sin NO \,.\, \sin Pp}{\sin NP};$$

$$\text{or } EO = \frac{R \,.\, Pp}{\sin NP};$$

but if L be taken for the mean extreme latitude the $Pp = \dfrac{L \,.\, \sin NP}{h}$

$$\therefore EO = \frac{R}{\sin NP} \times \frac{L \,.\, \sin NP}{h} = \frac{R \,.\, L}{h},$$

This is the mean extreme latitude stated in the GANITADHYÁYA multiplied by the radius and divided by the 2nd hypothenuse equals the true or rectified extreme latitude.— B. D.]

celestial latitude is in like manner an arc of a great circle (which passes through the ecliptic poles) intercepted between the ecliptic and the KSHEPA-VṚITTA.

The corrected declination [of any of the small planets and Moon] is the distance of the planet from the equinoctial in a circle of declination.

Precession of the equinox.

17. The point of intersection of the equinoctial and ecliptic circles is the KRÁNTI-PÁTA or intersecting point for declination. The retrograde* revolutions of that point in a KALPA amount to 30,000 according to the author of the SÚRYA-SIDDHÁNTA.

18. The motion of the solstitial points spoken of by MUNJÁLA and others is the same with this motion of the equinox: according to these authors its revolutions are 199,669 in a KALPA.

19. The place of the KRÁNTI-PÁTA, or the amount of the precession of the equinox determined through the revolutions of the KRÁNTI-PÁTA must be added to the place of a planet; and the declination then ascertained. The ascensional difference and periods of rising of the signs depend on the declination: hence the precession must be added to ascertain the ascensional difference and horoscope.

20. Thus the points of intersection of the ecliptic and the orbits of the Moon and other planets are the KSHEPA-PÁTAS, or intersecting points for the KSHEPA celestial latitude. The revolutions of the KSHEPA-PÁTAS are also contrary to the order of the signs, hence to find their latitudes, the places of the KSHEPA-PÁTAS must be added to the places of the planets (before found).

21. As the MANDA-SPASHTA planet (or the mean planet corrected by the 1st equation) and its ascending node revolve in the S'ÍGHRA-PRATIVṚITTA or 2nd excentric, hence the amount of the latitude is to be ascertained from (the place of) the MANDA-SPASHTA planet added to the node found by calculation.

* The motion of the KRÁNTI-PÁTA is in a contrary direction to that of the order of the signs.—L. W.

22. Or the amount of the latitude may be found from the SPASHTA planet added to the node which the S'ÍGHRA-PHALA 2nd equation is added to or subtracted from accordingly as it was subtractive or additive.*

As the Moon's node revolves in the concentric circle, the amount of the latitude, therefore, is to be found from the true place of the Moon added to the mean node.

23. The exact revolutions of the nodes of Mercury and Venus will be found by adding the revolutions of their S'ÍGHRA-KENDRAS to the revolutions of their nodes which have been stated [in the GAṆITÁDHYÁYA]: if it be asked why these smaller amounts have been stated, I answer, it is for greater facility of calculation. Hence their nodes which are found from their stated revolutions are to be added to the places of their S'ÍGHRA-KENDRAS [to get the exact places of the nodes].†

24. To find the KENDRA [of any of the planets] the place of the planet is subtracted from the S'ÍGHROCHCHA: then take

* [See the nodes on V. 11, and V. 13, 14, 15. —B. D.]

† [In all the original astronomical works, the sum of the PA'TA and S'ÍGHROCHCHA of Mercury and Venus, is assumed for their VIKSHEPA-KENDRA, and through this, their latitude is determined. But the latitude thus found would be at the place of their S'ÍGHROCHCHA and not at their own place, because their places are different from those of their S'ÍGHROCHCHAS. To remove this difficulty, BHA'SKRA'CHA'RYA writes. "The exact revolutions &c." But the difficulty arises in the supposition that, the earth is stationary in the centre of the universe and all the planets revolve round her, because we are then bound to grant that the mean places of Mercury and Venus are equal to that of the Sun, and hence their places will be different from those of their S'ÍGHROCHCHAS. But no inconvenience occurs in the supposition that, the Sun is in the centre of the universe and all the planets together with the earth revolve round him. For, in this case the places of the S'ÍGHROCHCHAS of Mercury and Venus are their own heliocentric places, and consequently the sum of the places of their S'ÍGHROCHCHAS and PA'TAS will be equal to the sum of their own places and those of their PA'TAS, that is to their VIKSHEPAKENDRA. For this reason, their latitude found through this, will be at their own places. Now, it is a curious fact that, the revolutions of the pátas of Mercury and Venus, stated in the original works, are such as ought to be mentioned when it is supposed that the Sun is in the middle of the universe and the planets revolve round him, and not when the Earth is supposed to be stationary in the centre of the universe. From this fact, we can infer that the original Authors of the Astronomical works knew that all the planets together with the Earth revolve round the Sun, and consequently they stated the smaller amounts of the revolutions of the PA'TAS of the Mercury and Venus. When this is the case, why is it supposed that all the planets revolve round the Earth, because the Spherics can more easily be understood by this supposition than by the other.—B. D.]

the KENDRA with the PA'TA added [to get the exact amount of the PÁTA or node] and let the place of the planet be added thereto, [we thus get the VIKSHEPA-KENDRA or the argument of the latitude of Mercury or Venus]. Therefore from the S'ÍGHROCHCHAS of these two planets with the PA'TAS added, their latitudes are directed by the ancient astronomers to be found.*

25 and 26. The PA'TAS or nodes of these two planets added to the S'ÍGHROCHCHRAS from which the true places of the planets have been subtracted, become SPASHṬA or rectified. It is the S'PASHTA-PA'TA which is found in the BHAGOLA (above described).

In the sphere of a planet, take the ecliptic above described as the concentric circle, to this circle the second excentric circle should be attached, as was explained before, and a circle representing the orbit of a planet (and which consequently would represent the real second excentric) should be also attached to the latter circle with the amount of latitude detailed for it. In this latter circle mark off the mean places of the nodes of the (superior) planets, and also mark in it the mean place of the nodes of Mercury and Venus added to their respective S'ÍGHRA-KENDRAS.†

27. Next the AHOṚATRA-VRITTAS or diurnal circles, must be made on both sides of the equinoctial [and parallel to it] at every or any degree of declination that may be required:—and they must all be marked with 60 GHATIS: The radius of the diurnal circle [on which the Sun may move on any day] is called DYUJYÁ.

Diurnal circles called AHOR'ATRA-VṚITTAS.

* [Let, $h =$ S'ÍGHROCHCHA or the place of 2d higher apsis.
$k =$ the S'ÍGHRA-KENDRA.
$p =$ the place of the planet.
$n =$ PA'TA or the place of the ascending node.
and $N. =$ the exact PA'TA.
then $k = h - p$; and $h = k + n = h. - p + n$;
$\because$ VIKSHEPA KENDRA or argument of latitude of Mercury or Venus $=$ $N. + p = h. - p + n + p = h + n$.—B. D.]

† [See the note on verses 13, 14 and 15 :—B. D.]

28. From the vernal equinox mark the 12 signs in direct order, and then let diurnal circles be attached at the extremity of each sign.

29. On either side of the equinoctial, three diurnal circles should be attached in the order of the signs: these again will answer for the three following signs.

The BHAGOLA has thus been described. This is to be known also as the KHECHARA-GOLA, the sphere of a planet.

30. Or in the plane of the ecliptic bind the orbits of Saturn and of the other planets with cross diameters to support them, but these must be bound below (within) the ecliptic in successive circles one within the other, like the circles woven one within the other by the spider.

31. Having thus secured the BHAGOLA on the axis or YASHṬI, after placing it within the hollow cylinders on which the KHAGOLA is to be fastened, make the BHAGOLA revolve:—it will do so freely without reference to the KHAGOLA as its motion is on the solid axis. The KHAGOLA and DṚIGGOLA remain stationary whilst the BHAGOLA revolves.

End of Chapter VI. on the construction of an armillary sphere.

CHAPTER VII.

Called TRIPRAS'NA-VÁSANÁ *on the Principles of the Rules for resolving the questions on time, space, and directions.*

The ascensional difference and its place.

1. The time called CHARA-KHUNDA or ascensional difference is found by that arc of a diurnal circle intercepted between the horizon and the six o'clock line. The sine of that arc is called the KUJYÁ in the diurnal circle: but, when reduced to relative

value in a great circle, it is called CHARAJYÁ or sine of ascensional difference.*

2. The horizon, as seen at the equator, or in a right sphere, is denominated in other places [to the north, or south of the equator] the UNMANDALA six o'clock line: but as the Sun appears at any place to rise on its own horizon, the difference between the times of the Sun's rising [at a given place and the equatorial region under the same meridian] is the ascensional difference.

Determination of the question when the CHARA correction is additive and when subtractive.

3. When the sun is in the northern hemisphere, it rises at any place (north of the equator) before it does to that on the equator: but it sets after it sets to that on the equator. Therefore the correction depending on the ascensional difference is to be subtracted at sunrise of a given place from the place of the planet [at sunrise at the equator] and to be added at sunset to the place of the planet [as found for the sunset at the equator].

4. When the Sun is in the southern hemisphere the reverse of this takes place, as the part of the UNMANDALA in that hemisphere lies below the horizon. The halves of the sphere north and south of the equinoctial are called the northern and southern hemispheres.

Cause of increase and decrease in length of days and nights.

5. [And it is in consequence of this ascensional difference that] the days are longer and the nights shorter (than they are on the

* [The times found by the arcs intercepted between the horizon and the six o'clock line, of the three diurnal circles attached at the end of the first 3 signs i. e. Aries, Taurus and Gemini are called the CHARA-KA'LAS or the ascensional differences of these signs, and the differences of these CHARA-KA'LAS are called the CHARA-KHANDAS of those three signs.

As, where the PALABHA is 5 digits or the latitude is nearly 22¼° north, the ascensional differences of the 3 first signs are 297, 541 and 642 ASUS, and the differences of those i. e. 297, 244 and 101 are the CHARA-KHANDAS of those signs.

These are again the CHARA-KHANDAS of the following three signs inversely i. e. 101, 244 and 297 ASUS.

Thus the CHARA-KHANDAS of the first six signs answer for the following six signs.—B. D.]

equator) when the Sun is in the northern hemisphere: and that the days are shorter and the nights longer when the Sun is in the southern hemisphere. For, the length of the night is represented by that arc of the diurnal circle below the horizon, and the length of the day by that arc above the horizon.

6. But at the equator the days and nights are always of the same length, as there is no UNMAṆḌALA there except the horizon [on the distance between which, the variation in the length of days and nights depends].

A circumstance of peculiar curiosity, however, occurs in those places having a latitude greater than 66° N. viz. than the complement of the Sun's greatest declination.

Determination of place and time of perpetual day and night.

7. Whenever the northern declination of the Sun exceeds the complement of the latitude, then there will be perpetual day for such time as that excess continued; and when the southern declination of the Sun shall exceed the complement of the latitude, then there will be perpetual night during the continuance of that excess. On MERU, therefore, day and night are each of half a year's length.

Place of MERU.

8. To the Celestial Beings [on MERU at the north pole] the equinoctial is horizon: so also is to the DAITYAS [at the south pole]. For, the northern and southern poles are situated respectively in their zeniths.

9. The Celestial Beings on MERU behold the Sun whilst he is in the northern hemisphere, always revolving above the horizon from left to right: but DAITYAS the inhabitants of the southern polar regions behold him whilst he is in the southern hemisphere revolving above their horizon from the right to the left.

Definition of the artificial day and night and the day and night of the PITṚIS.

10. Thus it is day whilst the Sun is visible, and night whilst he is invisible. As the determination of

night and day is made in regard to men residing on the surface of the Earth, so also is that of the PITṚIS or deceased ancestors who dwell on the upper part of the Moon.

The meaning of the fact stated by the astrological professors or SA'NHITIKAS.

11. As for the doctrine of astrologers, that it was day with the Gods at MERU whilst the Sun was in the UTTARÁYAṆA (or moving from the winter to the summer solstice) and night whilst the Sun was in the DAKSHIṆÁYANA (or moving from the summer to the winter solstice), it can only be said in defence of such an assertion, that it is day when the Sun is turned towards the day, and it is night when turned towards the night. Their doctrine has reference merely to judicial astrology and the fruits it foretells.

12. By the degrees by which the Sun proceeds in his northern course to the end of Gemini, he moves back from that sign: entering also the same diurnal circles in his descent as he did in his ascent. Is it not therefore that the Sun is visible in his descent to the Gods in the place where he was first seen by them in his ascent?

Length of the day of the PITṚIS.

13. The PITRIS reside on the upper part of the Moon and fancy the fountain of nectar to be beneath themselves. They behold the Sun on the day of our AMÁVÁSYÁ or new Moon in their zenith. That therefore is the time of their midday.

14. They (i. e. the PITṚIS) cannot see the Sun when he is opposite the lower part of the Moon: it is therefore, midnight with the PITRIS on the day of the PÚRṆIMÁ or full Moon. The Sun rises to them in the middle of the KRISHNA PAKSHA or dark half of the Moon, and sets in the middle of the S'UKLA PÁKSHA or light half of the Moon. This is clearly established from the context.

The explanation of a day of BRAHMA'.

15. As BRAHMÁ being at an immense distance from the Earth, always sees the Sun till the time of the PRALAYA or general deluge, and sleeps for the same time, therefore

the day and night of BRAHMA' are together of 2000 MAHA'YUGAS in length.

The time taken by each sign in rising above the horizon.

16. As the portion of the ecliptic which is more oblique than the other, rises and sets in a shorter time and that which is more upright takes a longer time in rising and setting, hence the times of rising of the several signs are various [even at the equatorial regions].

17. The (six) signs from Capricorn to Gemini or ascending signs which are inclined towards the south with their respective declinations whilst they rise even at the equator are still more inclined towards the south in the northern latitudes (on account of the obliquity of the starry sphere towards the south); hence they arise in still shorter times than they do at the equator.

18. At the equator, the [six] signs from Cancer or descending signs incline whilst they rise to the northerly direction, but they will have upright direction in consequence of the northern latitude, hence they rise in longer times [than they do at the equator.] The difference between the period of the rising of a sign in a given latitude, and at the equator under the same meridian, is equivalent to the CHARAKHAṆḌA of that sign.

19. Each quarter of the ecliptic rises in 15 GHAṬIS or 6 hours to those on the equator: and the 6 signs of the northern as well the 6 of the southern hemisphere appear to rise each in 12 hours or 30 GHAṬIS in every or any latitude.

20. The three signs from the commencement of Aries to the end of Gemini, i. e. the first quarter of the ecliptic, pass the UNMAṆḌALA in 15 GHAṬIS; but the horizon [of a place in north latitude] is below the UNMAṆḌALA, they therefore previously pass it in time less than 15 GHAṬIS by the CHARAKHAṆḌAS.

21. The three signs from the end of Virgo to the end of Sagittarius, i. e. the 3rd quarter of the ecliptic, pass the UNMAṆḌALA in 15 GHAṬIS; but they pass the horizon of a place

afterwards which is above the UNMAṆḌALA [in north latitude] in 15 GHAṬIS added to the CHARAKHAṆḌAS.

22. The three signs from the end of Gemini to the end of Virgo, i. e. the 2nd quarter of the ecliptic or those from the end of Sagittarius to the end of Pisces i. e. the 4th quarter of the ecliptic, pass the horizon in the time equal to the remainder of 30 GHAṬIS diminished by the time which the first or third quarter takes to pass the horizon respectively. For this reason, the times which the signs contained in the 1st and 4th quarters of the ecliptic, or ascending signs, and those contained in the 2nd and 3rd quarters, or descending signs take to pass the horizon at a given place are found by subtracting the CHARAKHAṆḌAS of the signs from and adding them to the times which those signs take in rising on the equator respectively.*

23. Having placed the 1st Aries in the horizon and set the sphere in motion, the tutor should show the above facts to the

* The times taken by the several signs of the ecliptic in rising at the equator and in northern latitudes will be seen from the following memo. according to the SIDDHANTA.

	Times of rising at equator in sidereal time.	Ascensional differences in 22° 30' north latitude sidereal time.	Times of rising in 22° 30' north latitude sidereal time.	
	ASUS.	ASUS.	ASUS.	
Aries,	1670	— 297	1373	These 3 and the last 3 signs take less time to rise in north latitude than at the equator.
Taurus,	1793	— 244	1549	
Gemini,	1937	— 101	1836	
Cancer,	1937	+ 101	2038	These 6 signs take a longer time to rise in north latitude than at the equator.
Leo,	1793	+ 244	2037	
Virgo,	1670	+ 297	1967	
Libra,	1670	+ 297	1967	
Scorpio,	1793	+ 244	2037	
Sagittarius,	1937	+ 101	2088	
Capricorn,	1937	— 101	1836	
Aquarius,	1793	— 244	1549	
Pisces,	1670	— 297	1378	

L. W.

pupils, that they may understand as well what has been explained as any other facts which have not been now mentioned.

24. In whatever time any sign rises above the horizon [in any latitude] the sign which is the 7th from it, will take exactly the same time in setting: as one half of the ecliptic is always above the horizon [in every latitude].

25. When the complement of latitude is less than 24° (i. e. than the extreme amount of the Sun's declination taken to be 24° by Hindu astronomers) then neither the rising periods of the signs, nor the ascensional differences and other particulars will correspond with what has been here explained. The facts of those countries (having latitudes greater than 66°) which are different from what has been explained on account of their totally different circumstances, are not here mentioned, as those countries are not inhabited by men.

Etymology of the word LAGNA.

26. That point of the ecliptic which is (at any time) on the eastern horizon is called the LAGNA or horoscope. This is expressed in signs, degrees, &c. reckoned from the first point of stellar Aries. That point which is on the western horizon is called the ASTA-LAGNA or setting horoscope. The point of the ecliptic on the meridian is called the MADHYA-LAGNA or middle horoscope (culminating point of the ecliptic).*

* [When the place of the horoscope is to be determined at a given time it is necessary at first to ascertain the height and longitude of the nonagesimal point from the right ascension of mid-heaven, and then by adding 3 signs to the longitude of the nonagesimal point, the place of the horoscope is found: but as this way for finding the place of the horoscope is very tedious, it has been determined otherwise in the SIDDHA'NTAS.

As, from the periods of risings of the 12 signs of the ecliptic which are determined in the Siddhantas, it is very easy to find the time of rising of any portion of the ecliptic and vice versa, we can find a portion of the ecliptic corresponding to the given time from sun-rise through the longitude of the Sun then determined and the given time. The portion of the ecliptic which can be thus found is evidently that portion of the ecliptic intercepted between the place of the Sun and the horizon. Therefore by adding this portion to the place of the Sun, the place of the horoscope is found. Upon this principle, the following common rule which is given in the SIDDHANTAS for finding the place of the horoscope is grounded.

Find first the true place of the Sun, and add to it the amount of the precession of the equinox for the longitude of the Sun. Then, from the longitude of the Sun, the sign of the ecliptic in which the Sun lies and the degrees of that sign

27. If when you want to find the LAGNA, the given GHAṬIS are SÁVANA-GHAṬIS, then they will become sidereal by finding the Sun's instantaneous place i. e. the place of the Sun for the hour given. The times

The reason for finding the exact place of the Sun at the time of question in order to find LAGNA.

which he has passed, and those which he has to pass, are known. Thus the degrees which the Sun has passed, and those which he has to pass, are called the BHUKTÁNS'AS and BHOGYÁNS'AS respectively. Now the time which the Sun requires to pass the BHOGYÁNS'AS is called the BHOGYA time, and is found by the following proportion.

If 30°
: the period of rising of the sign in which the Sun is
: : BHOGYÁNS'AS
: BHOGYA time.

In the same manner, the BHUKTA time can also be found through the BHUKTÁNSAS.

Now from the time at the end of which the horoscope is to be found, and which is called the ISHTA or given time, subtract the BHOGYA time just found, and from the remainder subtract the periods of risings of the next successive signs to that in which the Sun is as long as you can. Then at last you will find the sign, the rising period of which being greater than the remainder you will not be able to subtract, and which is consequently called the AS'UDDHA sign, or the sign incapable of being subtracted, and its rising period, AS'UDDHA rising. From this it is evident that the AS'UDDHA sign is of course on the horizon at the given time. The degrees of the AS'UDDHA sign which are above the horizon and therefore called the BHUKTA or passed degrees, are found as follows.

If the rising period of the AS'UDDHA sign
: 30°
: : the remainder of the given time
: the passed degrees of the AS'UDDHA sign.

Add to these passed degrees thus found, the preceding signs reckoned from the 1st point of Aries, and from the Sum, subtract the amount of the procession of the equinox. The remainder thus found will be the place of the horoscope from the stellar Aries.

If the time at the end of which the horoscope is to be found, be given before sun-rise, then find the BHUKTA, or passed time of the sign in which the Sun is, in the way above shown, and subtract it and the rising periods of the preceding signs from the given time. After this find the degrees of the AS'UDDHA sign corresponding to the remainder of the given time which will evidently be the BHOGYA degrees of the horoscope by proportion as shown above, and subtract the sum of the BHOGYA degrees of the horoscope, the signs the rising periods of which are subtracted and the BHUKTA degrees of the sign in which the Sun is from the Sun's place and the remainder thus found will be the place of the horoscope.

Thus we get two processes; one when the given time at the end of which the horoscope is to be found, is after sun-rise, and the other when that time is given before sun-rise, and which are consequently called KRAMA, or direct, and VYUTKRAMA or undirect processes respectively.

It is plain from this that if the place of the Sun and that of the horoscope be known, the given time from sun-rise at the end of which the horoscope is found can be known by making the sum of the BHOGYA time of the sign in which the Sun is and the BHUKTA time of the horoscope and by adding to this sum the rising periods of intermediate signs.—B. D.]

of rising of the signs which are sidereal must be subtracted from these GHAṬIS (of the question) reduced to a like denomination. When the hours of the question are already sidereal, there is no necessity for finding the sun's real place for that time.*

* [If it be asked whether the time at the end of which the horoscope is to be found is terrestrial or sidereal time; if it be terrestrial, how it is that you subtract from that the rising periods which are of different denomination on account of their being sidereal, and why the sun's instantaneous place i. e. the place determined for the hour given is used to ascertain the BHOGYA time, the given time is reckoned from sun-rise and the BHOGYA degrees of the sign in which the sun is, rise gradually above the horizon after sun-rise. Hence the BHOGYA degrees of the sign of the Sun's longitude, determined at the time of sun-rise, should be taken to find the place of the Horoscope, otherwise the place of the Horoscope will be greater than the real one. As for example, take the time from sun-rise, at the end of which the Horoscope is to be found, equal to 60 sidereal GHAṬIS and 44 ASUS when the Sun is in the vernal equinox at a place where the PALABHA is 5 digits or the latitude is 22°½ nearly, and ascertain the place of the Horoscope through the instantaneous place of the sun. Then, the place of the Horoscope thus found will be greater than the place of the Sun found at the time of next sun-rise, but this ought to be equal to it, and you will not be able to make this equal to the place of the Sun determined at the time of next sun-rise, unless you determine this through the place of the sun ascertained at sun-rise, and not through the Sun's instantaneous place. Hence it appears wrong to ascertain the place of the Horoscope through the Sun's instantaneous place. But the answer to this is as follows.

The GHAṬIS contained in the arc of the diurnal circle intercepted between that point of it where the Sun is, at a given time and the Horizon are the SÁVANA or terrestrial GHATIS, but the GHATIS contained in the arc of the diurnal circle intercepted between that point of it where the Sun was at the time of sun-rise and the Horizon are the sidereal, GHATIS. Thus it is plain from this that if the Sun's place determined at the time of sun-rise be given, the time between their place and the Horizon reckoned in the diurnal circle will evidently be the sidereal time and consequently the place of the Horoscope determined through this will be right. But if the instantaneous place of the Sun be given, the time given must be the SÁVANA time, because let the instantaneous place of the Sun be assumed for the Sun's place determined at the time of sun-rise, then the time between this assumed instantaneous place of the Sun and the Horizon, which is SÁVANA, will evidently be the sidereal time. Hence the fact as stated in the verse 27th is right.

Therefore if the Sun's instantaneous place and the place of the Horoscope be given, the time found through these will be the SÁVANA time, but if the place of the Horoscope and that of the Sun determined at the time of sun-rise be given, the time ascertained through these will be the sidereal time. And if you wish to find the SÁVANA time through the place of the Horoscope and that of the Sun determined at the time of the sun-rise assumed the sidereal time just found as a rough SÁVANA time and determined through this the instantaneous place of the Sun by the following proportion.

If 60 GHATIS
: Sun's daily motion
: : these rough SÁVANA GHATIS
: the Sun's motion relating to this time; and add then this result to the place of the Sun found at the time of sun-rise. The sum thus found will be the instantaneous place of the Sun nearly. Find the time again through this

Determination of latitudes in which different signs are always above and below the horizon.

28. In those countries having a north latitude of 69° 20′ the signs sagittarius and capricornus are never visible: and the signs gemini and cancer remain always above the horizon.

29. In those places having a northern latitude of 78° 15′, the four signs scorpio, sagittarius, capricornus, and aquarius are never seen, and the four signs taurus, gemini, cancer, and leo, always appear revolving above the horizon.

30. On that far-famed hill of gold MERU which has a latitude of 90° N. the six signs of the southern hemisphere never appear above the horizon and the six northern signs are always above the horizon.

An error of LALLA exposed.

31. LALLA has declared that when the ASUS of CHARA-KHANDA [in any latitude] are equal to the time which any sign takes to rise on the equator, then that sign will always remain visible above the horizon: but this assertion is without reason. Were it so, then in places having a latitude of 66°, the whole twelve signs of the ecliptic would always be visible, and would all appear at once on all occasions, as the times of their rising on the equator are equal to the ASUS of their CHARA-KHANDAS: but this is not the fact.

Another gross error of LALLA.

32. LALLA has also stated in his work on the sphere that where the north latitude is 66° 30′, sagittarius and capricornus are not visible, and also that in north latitude 75°, scorpio and aquarius are never there visible: but this also is an idle assertion. How, my learned friend, has he managed to make so gross and palpable an error of three degrees?*

instantaneous place of the Sun, and through this time ascertain the instantaneous place of the Sun. Thus you will get at last the exact SÁVANA time from sun-rise to the hour given by the repetition of this process. As the Sun is taken here for an example, you can find the SÁVANA time of any planet or any planetary time from the planet's rising to the hour given by the repetition of the aforesaid process.—B. D.]

* [BHÁSKARÁCHARYA means here that LALLA mentioning the degrees of latitudes, has committed a grand mistake in omitting 3 degrees, because he has

33. The altitude of the polar star and its zenith distance as found by observation, give respectively the latitude and the LAMBÁNSA or complement of the latitude. Or the zenith distance and altitude of the Sun at mid-day when on the equinoctial give the latitude and its complement.

34. The UNNATA the time found in that arc of the diurnal circle which is intercepted between the eastern or western horizon and the planet above it, is SÁVANA. This is used in finding the shadow of the planet. The sine of the UNNATA which is oblique, like the AKSHA-KARṆA, by reason of the latitude, is called CHHEDAKA and not S'ANKU because it is upright.*

35. In order to find the shadow of the Moon, the UDITA (the time elapsed from the rising of a planet) which has been found by some astronomers by means of repeated calculation is erroneous, for the UDITA, (found by repeated calculation) is not SÁVANA. The labour of the astronomer that does not thoroughly understand mathematics as well the doctrine of the

stated in his work that sagittarius and capricornus are always visible in a place bearing a latitude 66° 30′, and scorpio and aquarius at 75° N., whereas this is not the case, those signs are always visible in the places bearing the latitudes 69° 30′ and 78° 15′ respectively as shown in the verses 28 and 29.—B. D.]

* [When the Sun is above the Horizon, the shadow caused by a gnomon 12 digits, high, is called the Sun's shadow according to the S'IDDHANTA languages and having at first determined the sine of the Sun's altitude and that of it complement through his UDITA time, astronomers ascertained this by the following proportion.

As the sine of the Sun's altitude
: the sine of its complement
: : gnomon of 12 digits
: the shadow caused by the gnomon.

Thus they determine the shadow of all planets, Moon, &c., and that of the fixed stars. Though the light of the five small planets, Mars, &c., and the fixed stars is not so brilliant, like that of the Sun and Moon, as to make their shadow visible, yet it is necessary to determine the shadow of any heavenly body in order to know the direction in which the body may be. Because, if the length and direction of the shadow of the body be known, the direction in which it is can be ascertained by spreading a thread from the end of its shadow through that of the gnomon. For, if you will fix a pipe in the direction of the thread thus spread, you will see through that pipe the body whose shadow is used here.

The time given for determination of any planet's shadow must be the SÁVANA time, because it is necessary to determine the degrees of altitude of a planet to know its shadow, and the degrees can be determined through the time contained in that arc of the diurnal circle intercepted between the planet and horizon. But the time contained in this arc cannot be other than the SÁVANA time.—B. D.]

sphere, in writing a book of instruction on the science is utterly futile and useless.*

Determination of S'ANKU and DṚIGJYÁ.

36. The degrees of altitude are found in the DṚINMANḌALA or vertical circle, being the degrees of elevation in it above the horizon; the degrees of zenith distance are (as their name imports) the degrees in the same circle by which the object is distant from the zenith or mid-heaven of the observer: the S'ANKU is the sine of the degrees of altitude: and the DṚIGJYÁ is the sine of the zenith distance.

Of SAMA-S'ANKU, KOṆA-S'ANKU and MADHYA-S'ANKU.

37. When the Sun in his ascent arrives at the prime vertical, the S'ANKU found at the moment is the SAMA-S'ANKU: the S'ANKUS found at the moments of his passing the KOṆA-VṚITTA and the meridian are respectively termed the KOṆA-S'ANKU and MADHYA-S'ANKU.

Reason of the correction of parallax to the sine of altitude.

38. One-half of the vertical circle in which a planet is observed should be visible, but only one-half less the portion opposite the radius of the Earth is visible to observers on the surface of the Earth. Therefore $\frac{1}{15}$ part of the daily motion of the planet observed is to be subtracted from the sine of altitude or from the S'ANKU to find the shadow: [inasmuch as that amount is concealed by, or opposite to, the Earth].

The sine of amplitude and the UDAYASTA-SUTRA.

39. The AGRÁ (the sine of amplitude) is the sine of the arc of the horizon intercepted between the prime vertical and the planet's diurnal circle in the east or west i. e. between

* [In order to determine the Moon's shadow at a given time at full moon, some astronomers find her UDITA time i. e. the time elapsed from her rising to the hour given by the repeated calculation, through her instantaneous place and the place of the horoscope determined at the given hour. But they greatly err in this, because the time thus found will not be the S'AVANA time and consequently they cannot use this in finding the Moon's shadow. Their way for finding the UDITA time by the repeated calculation would be right, then only if the given place of the Moon would be such as found at the time of her rising and not her instantaneous place. Because her UDITA time found through her instantaneous place becomes S'AVANA at once without having a recourse to the repeated calculation, as it is shown in the note on the verse 27 of this Chapter.—B D.]

the east or west point of the horizon, and the point of the horizon at which the planet rises or sets. The line connecting the points of the extremities of the east and west AGRÁ is called the UDAYÁSTA-SUTRA, the line of rising and setting.

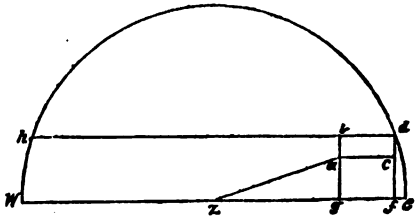
40. The S'ANKU-TALA or base of the S'ANKU stretches during the day to the south of the UDYÁSTA-SUTRA; because the diurnal circle have during the day a southern inclination (in northern latitude) above the horizon. But, below the horizon at night, the base lies to the north of the UDAYÁSTA-SUTRA as then the diurnal circles incline to the north. The S'ANKU-TALA's place has thus been rightly defined.

41. The S'ANKU-TALA lies to the south of the extreme point of AGRÁ when that AGRÁ is north and when the AGRÁ is south, the S'ANKU-TALA lies still to the south of it. The difference and sum of the sine of amplitude and S'ANKU-TALA has been denominated the BÁHU or BHUJA; it is the sine of the degrees lying between the prime vertical and the planet on the plane of the horizon.

42. [Taking this BÁHU as one side of a right-angled triangle.] The sine of the zenith distance being the hypothenuse then the third side or the KOṬI being the square root of the difference of their squares will be found: it is an east and west portion of the diameter of the prime vertical.*

I now propose to explain the triangles which are created by reason of the Sun's varying declination: and shall then proceed to explain briefly also the latitudinal triangles or those created by different latitudes. [The former are called KRÁNTI-KSHETRAS and the latter AKSHA-KSHETRAS.]

* Vide accompanying diagram.
a being place of the Sun: *d* its place of rising in the horizon: *d h* the UDAYA'STA-SÚTRA *d f* the AGRA': *a b* the S'ANKU-TALA: then *a g* is the BA'HU and the triangle *a z g* is the one here represented to.—L. W.



43. In the 1st triangle of declination.

1st. The sine of declination = BHUJA or base,
the radius of diurnal circle corresponding with the declination above given = KOṬI or perpendicular,
and radius of large circle = hypothenuse.

2nd. Or in a right sphere.
The sine of 1, 2 or 3 signs = hypothenuse:
The declination of 1, 2 or 3 signs in six o'clock line = BHUJAS.

44. Sines of arcs of diurnal circles corresponding with the declination above given = KOṬIS.

Those sines being converted into terms of a large circle: and their arcs taken, they will then express the times in ASUS which each sign of the ecliptic takes in rising at the equator i. e. the right ascensions of those signs or the LANKODAYAS, that is the 2nd will be found when the 1st is subtracted from two found conjointly, and the 3rd will be found when the sum of the 1st and 2nd is subtracted from three found conjointly.

45. In the right-angled triangle formed by the S'ANKU or gnomon when the Sun is on the equinoctial.*

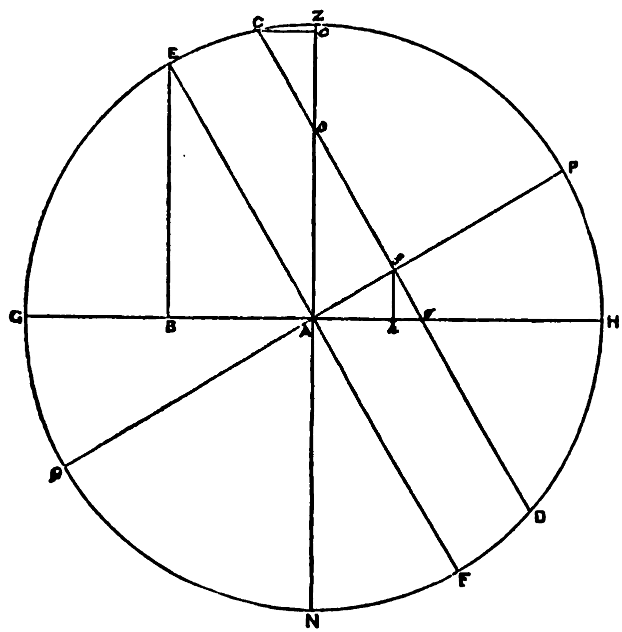
Triangles arise from latitude.

1st. The S'ANKU of 12 digits = the KOṬI.
The PALABHÁ or the shadow of S'ANKU or gnomon = the BHUJA
and the AKSHA-KARṆA = the KARṆA or hypothenuse
or 2nd. The sine of latitude = BHUJA.
The sine of co-latitude = KOṬI
and radius = hypothenuse
This triangle is found in the plane of the meridian.

[* The right angle triangles stated in the five verses from 45 to 49, are clearly seen by fastening some diametrial threads within the armillary sphere. As

46. Or the sine of declination reckoned on the UNMANḌALA from the east and west line $\}$ = KOṬI.

KUJYÁ, the sine of ascensional difference in the diurnal circle of the given day $\}$ = BHUJA.



Let Z G N H be the meridian of the given place, G A H the diameter of the horizon, Z the Zenith, P and Q the north and south poles, E A F the diameter of the equinoctial, P A Q that of the six o'clock line, O *f* D that of one of the diurnal circles, and E B, *f h* the perpendiculars to G H. Then it is clear from this that

Z E or H P = the latitude,
A B = the sine of it,
E B = the co-sine of it,
A *f* = the declination of a planet revolving in the diurnal circle whose diameter is O D,
and ∴ A *g* = the AGRA or the sine of amplitude,
f g = the KUJYA',
A *e* = the SAMA-SA'NKU or the sine of the planet's altitude when it reaches the prime vertical.
e g = the TADDHṚITI,
e f = the TADDHṚITI—KUJYA',
f h = the UNMANḌALA S'ANKU or the sine of the planet's altitude when it reaches the six o'clock line,
A *h* = the AGRA'DI-KHANḌA or the 1st portion of the sine of amplitude,
and *h g* = the AGRA'GRA-KHANḌA or the 2nd portion of the sine of amplitude;

The sine of amplitude in the horizon = hypothenuse
This is a well known triangle.

47. Or the SAMA S'ANKU in the prime vertical being } = KOṬI
The sine of amplitude = BHUJA
The TADDHṚITI in the diurnal circle = hypothenuse
Or
Taking the sine of declination = BHUJA
and the SAMA-S'ANKU = hypothenuse
TADDHṚITI *minus* KUJYÁ = KOṬI.

48. The UNMANḌALA S'ANKU being = BHUJA
The sine of declination will then be = hypothenuse
And AGRADI KHANDA or 1st portion of the sine of amplitude will be } = KOṬI

Therefore, with the exception of the first and last the other six triangles stated in the verses are these in succession. A E B, A gf, A eg, A ef, A fh and gfh and the first triangle you will get by dividing the three sides of the triangle A E B by $\frac{\text{E B}}{12}$ and for the last see the note on the verse 49.

It is clear from the above described diagram that all of these triangles are similar to each other and consequently they can be known by means of proportion if any of them be known.

The SIDDHÁNTIS, having thus produced several triangles similar to these original by fastening the threads within the armillary sphere, find answers of the several questions of the spherical trigonometry. Some problems of the spherical trigonometry can be solved with greater facility by this SIDDHANTA way than the trigonometrical way. As

Problem. The zenith distances of a star when it has reached the prime vertical and the meridian at a day in any place are known, find the latitude in the place.

The way for finding the answer of this problem according to the SIDDHANTA is as follows.

Draw C c ⊥ A Z, (See the proceeding diagram) then C c e will be a latitudinal triangle.

Now, let a = C c, the sine of zenith distance,
b = A c, the co-sine of Z c,
c = A e, the SAMA-S'ANKU,
and x = the latitude.

Then $Ce = \sqrt{a^2 + (b-c)^2}$,
and $Ce : Cc :: AE : AB$,
or $\sqrt{a^2 + (b-c)^2} : a :: \text{rad} : \sin x$;

$$\therefore \sin x = \frac{a \times \text{Rad}}{\sqrt{a^2 + (b-c)^2}}.\text{—B. D.]}$$

Or

Making the UNMANDALA SANKU = KOTI

the AGRÁGRA-KHANDA or 2nd portion of the sine of amplitude is } = BHUJYA

the KUJYA then becomes = hypothenuse

49.* The S'ANKU being = KOTI

and the S'ANKU-TALA = BHUJA

Then the CHHEDAKA or HRITI = hypothenuse

Those who have a clear knowledge of the spherics having thus immediately formed thousands of triangles should explain the doctrine of the sphere to their pupils.

End of Chapter VII. on the principles of the rules for resolving the questions on time, space and directions.

CHAPTER VIII.

Called GRAHAṆA VÁSANÁ.

In explanation of the cause of eclipses of the Sun and Moon.

The cause of the directions of the beginning and end of the solar eclipse.

1. The Moon, moving like a cloud in a lower sphere, overtakes the Sun [by reason of its quicker motion and obscures its shining disk by its own dark body :] hence it arises that the western side of the Sun's disk is first obscured, and that the eastern side is the last part relieved from the Moon's dark body : and to some places the Sun is eclipsed and to others is not eclipsed (although he is above the horizon) on account of their different orbits.

* This triangle differs from the 1st of the 47th verse only in this respect that the base of the triangle in the 47th verse is equal to the sine of the *whole amplitude* while the base found when the Sun is not in the prime vertical, will always be more or less than the sine of amplitude and is therefore generally called SANKUTÀLA.—L. W.

2. At the change of the Moon it often so happens that an observer placed at the centre of the Earth, would find the Sun when far from the zenith, obscured by the intervening body of the Moon, whilst another observer on the surface of the Earth will not at the same time find him to be so obscured, as the Moon will appear to him [on the higher elevation] to be depressed from the line of vision extending from his eye to the Sun. Hence arises the necessity for the correction of parallax in celestial longitude and parallax in latitude in solar eclipses in consequence of the difference of the distances of the Sun and Moon.

The cause of the parallax in longitude and that in latitude.

3. When the Sun and Moon are in opposition, the Earth's shadow envelopes the Moon in darkness. As the Moon is actually enveloped in darkness, its eclipse is equally seen by every one on the Earth's surface [above whose horizon it may be at the time]: and as the Earth's shadow and the Moon which enters it, are at the same distance from the Earth, there is therefore no call for the correction of the parallax in a lunar eclipse.

The reason of the correction of parallax not being necessary in lunar eclipses.

4. As the Moon moving eastward enters the dark shadow of the Earth: therefore its eastern side is first of all involved in obscurity, and its western is the last portion of its disc which emerges from darkness as it advances in its course.

The cause of the directions of the beginning and end of the lunar eclipse.

5. As the Sun is a body of vast size, and the Earth insignificantly small in comparison: the shadow made by the Sun from the Earth is therefore of a conical form terminating in a sharp point. It extends to a distance considerably beyond that of the Moon's orbit.

6. The length of the Earth's shadow, and its breadth at the part traversed by the Moon, may be easily found by proportion.

In the lunar eclipse the Earth's shadow is northwards or southwards of the Moon when its latitude is south or north. Hence the latitude of the Moon is here to be supposed inverse (i. e. it is to be marked reversly in the projection to find the centre of the Earth's shadow from the Moon.)

The determination of the coverer in the eclipse of the Sun and Moon.

7. As the horns of the Moon, when it is half obscured form very obtuse angles: and the duration of a lunar eclipse is also very great, hence the coverer of the Moon is much larger than it.

8. The horns of the Sun on the contrary when half of its disc is obscured form very acute angles: and the duration of a solar eclipse is short: hence it may be safely inferred that the dimensions of the body causing the obscuration in a solar eclipse are smaller than and different from the body causing an eclipse of the Moon.*

9. Those learned astronomers, who, being too exclusively devoted to the doctrine of the sphere, believe and maintain that RÁHU cannot be the cause of the obscuration of the Sun and Moon, founding their assertions on the above mentioned contrarieties, and differences in the parts of the body first obscured, in the place, time, causes of obscuration &c. must be admitted to assert what is at variance with the SANHITÁ, the VEDAS and PURÁṆAS.

10. All discrepancy, however, between the assertions above referred to and the sacred scriptures may be reconciled by understanding that it is the dark RÁHU which entering the Earth's shadow obscures the Moon, and which again entering the Moon (in a solar eclipse) obscures the Sun by the power conferred upon it by the favour of BRAHMÁ.

* [Had the Sun's coverer been the same with that of the Moon, his horns, when he is half eclipsed, would have formed, like those of the Moon obtuse angles. For the apparent diameters of the Sun and Moon are nearly equal to each other. Or the Moon when it is half eclipsed would have represented its horns, like those of the Sun, forming acute angles, if its coverer had been the same with that of the Sun. But as this is not the case, the coverer of the Moon is, of course, different and much larger than that of the Sun.—B. D.]

11. As the spectator is elevated above the centre of the earth by half its diameter, he therefore sees the Moon depressed from its place [as found by a calculation made for the centre of the Earth]. Hence the parallax in longitude is calculated from the radius of the Earth, as is also the parallax in latitude.

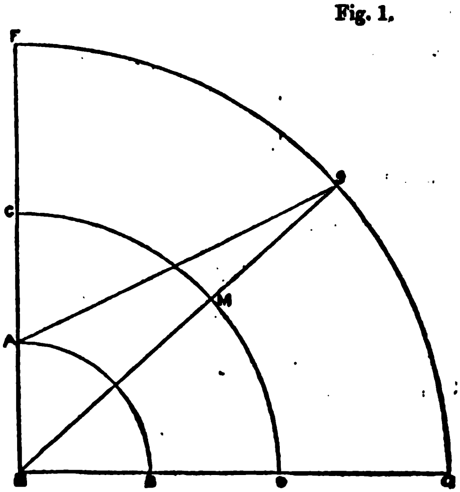
What is the cause of parallax, and why it is calculated from the radius of the Earth.

12. Draw upon a smooth wall, the sphere of the earth reduced to any convenient scale, and the orbits of the Moon and Sun at proportionate distances: next draw a transverse diameter and also a perpendicular diameter to both orbits.*

Construction of diagram to illustrate the cause of parallax.

13, 14 and 15. Those points of the orbits cut by this diameter are on the (rational) horizon. And the point above

Fig. 1.



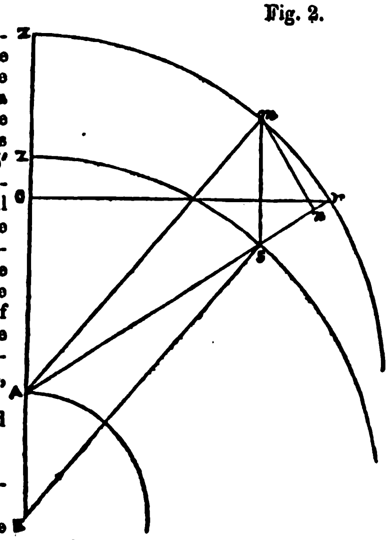
* In Fig. 1, let E be the centre of the earth; A a spectator on her surface; C D, F G the vertical circles passing through the Moon M, and the Sun S; D, G the points of the horizon cut by the vertical circles C D, F G; and C, the zenith in the Moon's sphere, and F in that of the Sun. Now, let E M S be a line drawn from the centre of the Earth to the Sun in which the Moon lies always at the time of conjunction, and A S the vision line drawn from the spectator A to the Sun. The distance at which the Moon appears depressed from the vision line in the vertical circle is her parallax from the Sun.

When the Sun reaches the zenith F, it is evident that the Moon also will then be at C and the vision line, and the line drawn from the centre of the Earth will be coincident. Hence there is no parallax in the zenith.

Thus the parallax of the Moon from the Sun in the vertical circle is here shown by means of a diagram which becomes equal to the difference between the parallaxes of the Sun and Moon separately found in the vertical circle as stated by BHA'SKARA'CHA'RYA in the chapter on eclipses in the commentary VA'SANA'BHA'SHYA and the theories and methods are also given by him on the parallaxes of the Sun and Moon. This parallax in the vertical circle which arises from the zenith distance of the planet is called the common parallax or the parallax in altitude.

cut by the perpendicular diameter will represent the observer's zenith: Then placing the Sun and Moon with their respective zenith distances [as found by a proportional scale of sines and arcs,] let the learned astronomer show the manner in which

Fig. 2.



As in Fig. 2, let A be a spectator on the earth's surface; Z the zenith; and Z S the vertical circle passing through the planet S: Let a circle Z' m r be described with centre A and radius E S which cuts the lines A Z and A S produced in the points Z' and r: Let a line s m be drawn parallel to E Z, then the arc Z' m will be equal to the arc Z S. Now the planet S seen from E has a zenith distance Z S and from A, a zenith distance Z' r greater than Z S or Z' m by the arc m r, hence the apparent place r of the planet is depressed by m r in the vertical circle. This arc m r is therefore the common parallax of the planet, which can be found as follows,

Draw m n perpendicular to A r and r o to A Z and let P = E S or A r;
h = E A or m S;
p = m r the parallax;
d = Z S or Z' m the true zenith distance of the planet;
and $\therefore d + p$ = Z' r the apparent zenith distance of the planet,
Then m n = sin p and r o = sin $(d + p)$.
Now by similar triangles A r o, S m n.

$$A\,r : r\,o = S\,m : m\,n,$$

$$\text{or } R : \sin(d + p) = h : \sin p;$$

$$\therefore \sin p = \frac{h \times \sin(d + p)}{R}.$$

Hence, it is evident from this that when the sin $(d + p) = R$ or $d + p = 90^\circ$, then the parallax will be greatest and if it be denoted by P,

$$\sin P = h \text{ and } \therefore \sin p = \frac{\sin P \times \sin(d + p)}{R}$$

Now, the parallax is generally so small that no sensible error is introduced by making $\sin p = p$ and $\sin P = P$;

$$\therefore\ p = \frac{P \times \sin(d + p)}{R};$$

Again, for the reason just mentioned sin d is assumed for sin $(d + p)$ in the SIDDHÁNTAS,

$$\therefore p = \frac{P . \sin d}{R}$$

that is, the common parallax of a planet is found by multiplying the greatest parallax by the sine of the zenith distance and dividing the product by the radius.—B. D.]

the parallax arises. [For this purpose] let him draw one line passing the centre of the earth to the Sun's disc: and another which is called the DṚIKSÚTRA or line of vision, let him draw from the observer on the Earth's surface to the Sun's disc. The minutes contained in the arc, intercepted between these two lines give the Moon's parallax from the Sun.

16. (At the new Moon) the Sun and Moon will always appear by a line drawn from the centre of the earth to be in exactly the same place and to have the same longitude: but when the Moon is observed from the surface of the Earth in the DṚIKSÚTRA or line of vision, it appears to be depressed, and hence the name LAMBANA, or depression, for parallax.

17. (When the new Moon happens in the zenith) then the line drawn from the Earth's centre will coincide with that drawn from its surface, hence a planet has no parallax when in the zenith.

Now on a wall running due north and south draw a diagram as above prescribed; [i. e. draw the Earth, and also the orbits of the Sun and Moon at proportionate distances from the Earth, and also the diameter transverse and perpendicular, &c.]

18. The orbits now drawn, must be considered as DṚIKSHEPA-VṚITTAS or the azimuth circles for the nonagesimal. The sine of the zenith distance of the nonagesimal or of the latitude of the zenith is the DṚIKSHEPA of both the Sun and Moon.

19. Mark the nonagesimal points on the DṚIKSHEPA-VṚITTAS at the distance from the zenith equal to the latitude of the points. From these two points (supposing them as the Sun and Moon) find as before the minutes of parallax in altitude. These minutes are here NATI-KALÁS, i. e. the minutes of the parallax in latitude of the Moon from the Sun.

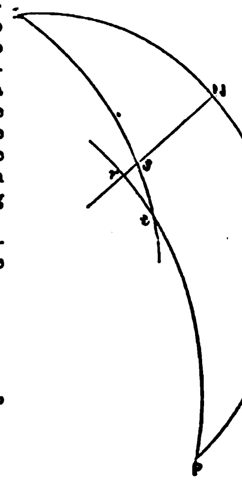
20. The difference north and south between the two orbits i. e. the measure of their mutual inclination, is the same in every part of the orbit as it is in the nonagesimal point, hence this difference called NATI is ascertained through the DṚIKSHEPA or the sine of the zenith distance of the nonagesimal.*

[* When the planet is depressed in the vertical circle, its north and south

21. The amount by which the Moon is depressed below the Sun deflected from the zenith [at the conjunction] wherever it be, is the east and west difference between the Sun and Moon in a vertical circle.*

distance from its orbit caused by this depression is called NATI or the parallax in latitude.

Fig. 3.



As, in Fig. 3, let Z be the zenith; N the nonagesimal; Z N P its vertical circle; N s r the ecliptic; P its pole; Z s t the vertical circle passing through the true place S and the depressed or apparent place t of the Sun; P t r a secondary to the ecliptic passing through the apparent place t of the Sun; then s r is the SPASHṬA LAMBANA or the parallax in longitude and t r the NATI or the parallax in latitude which can be found in the following manner according to the SIDDHÁNTAS.

Let Z N be the zenith distance of the nonagesimal and Z S that of the Sun; then by the triangles Z N S, t s r

$$\sin ZS : \sin ZN = \sin st : \sin rt,$$

$$\therefore \sin rt = \frac{\sin st \times \sin ZN}{\sin ZS};$$

Now, s t is taken for sin s t, and r t for sin r t, on account of their being very small

$$\therefore rt = \frac{st \times \sin ZN}{\sin ZS};$$

but according to the SIDDHÁNTAS

$$st = \frac{P \,.\, \sin ZS}{R} \text{ (see the preceding note)..} \quad (1)$$

$$\therefore rt = \frac{P \,.\, \sin ZN}{R} \text{ ....} \quad (2)$$

that is, the NATI is found by multiplying the sine of the latitude of the nonagesimal by the greatest parallax and dividing the product by the radius.

It is clear from this that the north and south distance from the Sun depressed in the vertical circle to the ecliptic wherever he may be in it, becomes equal to the common parallax at the nonagesimal, and hence the NATI is to be determined from the zenith distance of the nonagesimal.

For this reason, by subtracting the NATI of the Sun from that of the Moon, which are separately found in the way above mentioned, the parallax in latitude of the Moon from the Sun is found: and this becomes equal to the difference between the mean parallaxes of the Sun and Moon at the nonagesimal. The same fact is shown by BHÁSKARÁCHÁRYA through the diagrams stated in the verses 12th &c.

At the time of the eclipse as the latitude of the Moon revolving in its orbits is very small, the Moon, therefore, is not far from the ecliptic; and hence the parallax in longitude and that in latitude of the Moon is here determined from her corresponding place in the ecliptic, on account of the difference being very small.—B. D.]

* [According to the technicality of the Siddhantas, the distance taken in any circle from any point in it, is called the east and west distance of the point, and

22. For this reason, the difference is two-fold, being partly east and west, and partly north and south. And the ecliptic is here east and west, and the circle secondary to it is north and south. (It follows from this, that the east and west difference lies in the ecliptic, and the north and south difference in the secondary to it.)

23. The difference east and west has been denominated LAMBANA or parallax in longitude, whilst that running north and south is parallax in latitude.

24. The parallax in minutes as observed in a vertical circle, forms the hypothenuse of a right angle triangle, of which the NATI-KALÁ or the minutes of the parallax in latitude form one of the sides adjoining the right angle then the third side found by taking the square-root of the difference of the squares of the two preceding sides will be SPHUTA-LAMBANA-LIPTÁ or the minutes of the parallax in longitude.*

25. The amounts in minutes of parallax in a vertical circle may be found by multiplying the sine of the Sun's zenith distance of the minutes of the extreme or horizontal parallax and dividing the product by the radius. Thus the NATI will be found from the DṚIKSHEPA or the sine of the nonagesimal zenith distance.†

26. The extreme or horizontal parallax of the Moon from the Sun amounts to $\frac{1}{15}$ part of the difference of the Sun's and Moon's daily motion. For $\frac{1}{15}$ part of the YOJANAS, the distance of which any planet traverses per diem (according to the SIDDHÁNTAS) is equal to the Earth's radius.

27. The minutes of the parallax in longitude of the Moon from the Sun divided by the difference in degrees of the daily

the distance taken in the secondary to that circle from the same point, is called the north and south distance of that point.—B. D.]

* [See Fig. 3, in which by assuming the triangle $r\,s\,t$ as a plane right-angled triangle, $r\,t$ = base, $s\,t$ = hypothenuse and $s\,r$ = perpendicular, and therefore $s\,r = \sqrt{s\,t^2 - r\,t^2}$.—B. D.]

† [This is clear from the equations (1) and (2) shown in the preceding large note.—B. D.]

motions of the Sun and Moon will be converted into GHAṬIS [i. e. the time between the true and apparent conjunction].*

If the Moon be to the east [of the nonagesimal], it is thrown forward from the Sun, if to the west it is thrown backward (by the parallax).

28. And if the Moon be advanced from the Sun, then it must be inferred that the conjunction has already taken place by reason of the Moon's quicker motion; if depressed behind the Sun, then it may be inferred that the conjunction is to come by the same reason.

Hence the parallax in time, if the Moon be to the east [of the nonagesimal] is to be subtracted from the end of the TITHI or the hour of ecliptic conjunction, and to be added when the Moon is to the west [of the nonagesimal].

29. The latitude of the Moon is north and south distance between the Sun and Moon, and the NATI also is north and south. Hence the S'ARA or latitude applied with the NATI or the parallax in latitude, becomes the apparent latitude (of the Moon from the Sun).

VALANA or variation (of the ecliptic).

[The deviation of the ecliptic from the eastern point (in reference to the observer's place) of a planet's disc, situated in the ecliptic is called the VALANA or variation (of the ecliptic). It is evident from this, that the variation is equivalent to the arc which is the measure of the angle formed by the ecliptic and the secondary to the circle of position at the planet's place in the ecliptic. It is equal to that arc also, which is the

* It is clear from the following proportion.
If difference in minutes of daily motions of Sun and Moon.
: 60 GHATIS — what will
: : given LAMBANA—KA'LAS or minutes of the parallax give;

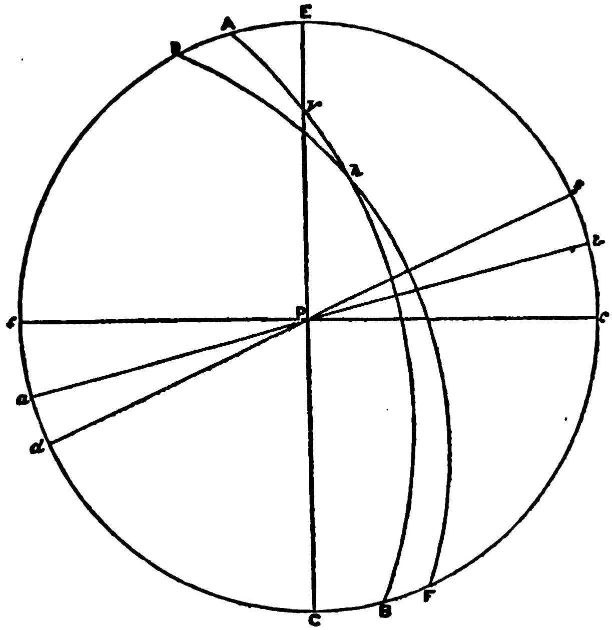
or $\dfrac{60 \times \text{given minutes of the parallax}}{\text{diff. in minutes of Sun's and Moon's motions}} = \dfrac{\text{given minutes of the parallax}}{\text{diff. in degrees of Sun's and Moon's motions}}$ = acceleration or delay of conjunction arising from parallax.—L. W.

measure of the angle at the place of the planet in the ecliptic formed by the circle of position and the circle of latitude. It is very difficult to find it at once. For this reason, it is divided into two parts called the ÁKSHA-VALANA (latitudinal variation) and the ÁYANA-VALANA (solstitial variation). The ÁKSHA-VALANA is the arc which is the measure of the angle formed by the circle of position, and the circle of declination at the place of the planet in the ecliptic, and the ÁYANA-VALANA is the arc which is the measure of the angle formed by the circle of declination and the circle of latitude. This angle is equivalent to the angle of position. From the sum or difference of these two arcs, the arc which is the measure of the angle formed by the circle of position and the circle of latitude is ascertained, and hence it is sometimes called the S'PASHṬA-VALANA or rectified variation.

Now, according to the phraseology of the SIDDHÁNTAS, the point at a distance of 90° forward from any place in any circle is the east point of that place, and the point at an equal distance backwards from it is the west point. And, the right hand point, 90° distant from that place, in the secondary to the former circle, is the south point, and the left hand point, is the north point. According to this language, the deviation of the east point of the place of the planet in the ecliptic, from the east point in the secondary to the circle of position at the planet's place, is the VALANA. But the secondary to the circle of position will intersect the prime vertical at a distance of 90° forward from the place of the planet, and hence the deviation of the east point in the ecliptic from the east point in the prime vertical is the VALANA or variation, and this results equally in all directions. When the east point in the ecliptic is to the north of the east point in the prime vertical, the variation is north, if it be to the south, the variation is south.

The use of the VALANA is this that, in drawing the projections of the eclipses, after the disc of the body which is to be eclipsed is drawn, and the north and south and the east and west lines are also marked in it, which lines will, of course, represent the circle of position and its secondary, the direction of the line representing the ecliptic in the disc of the body can easily be found through the VALANA. This direction being known, the exact directions of the beginning, middle and the end of the eclipse can be determined. But as the Moon revolves in its orbit, the direction of its orbit, therefore, is to be found. But the method for finding this is very difficult, and consequently instead of doing this, Astronomers determined the direction of the ecliptic, by means of the Moon's corresponding place in it and then ascertain the direction of the Moon's orbit.

The VALANA will exactly be understood by seeing the following diagram



Let E P C be the ecliptic, P the place of the planet in it, A *h* B the equinoctial, V the vernal equinox, D *h* F the prime vertical, *h* the point of intersection of the prime vertical and the equinoctial, hence h the east or west point of the horizon and D h equivalent to the NATA which is found in the V. 36. Again, let c P c, a P b and d P f be the circles of latitude, declination and position respectively passing through the place of the planet in the ecliptic.

Then,

the arc f b which is the measure of $\angle$ b P f = the ÁKSHA-VALANA :

the arc b c .... $\angle$ c P b = the ÁYANA-VALANA :

and the arc f c .... $\angle$ c P f = the SPASHṬA-VALANA.

Or according to the phraseology of the SIDDHÁNTAS

E the east point of P in the ecliptic;

A .... the equinoctial;

D .... the prime vertical;

hence,

the distance from D to A or arc D A or f b = the ÁKSHA-VALANA :

.... A to E or arc A E or b c = the ÁYANA-VALANA :

and .... D to E or arc D E or f c = the SPASHṬA-VALANA or rectified variation.

These arcs can be found as follows

Let, l = longitude of the planet,

e = obliquity of the ecliptic,

d = declination of the planet,

L = latitude of the place,

n = NATA,

x = ÁYANA-VALANA,

y = AKSHA-VALANA,

and Z = rectified VALANA.

Then, in the spherical triangle A V E,

$$\sin \text{E A V} : \sin \text{A V E} = \sin \text{E V} : \sin \text{A E},$$

or
$$\cos d : \sin e = \cos l : \sin x,$$

$\therefore$ sin x or sine of the ÁYANA-VALANA $= \dfrac{\sin e \,.\, \cos l\,.}{\cos d}$ (A)

See V. 32, 33, 34.

This VALANA is called north or south as the point E be north or south to the point A.

And, in the triangle A h D.

$$\sin D\,A\,h : \sin A\,h\,D = \sin D\,h : \sin D\,A\,;$$

here, $\sin D\,A\,h = \sin E\,A\,V = \cos d,$

$\sin A\,h\,D = \sin L,$

and $\sin D\,h = \sin n,$

$\therefore$ $\cos d : \sin L = \sin n : \sin y,$

$\therefore$ sin y or sine of the ÁKSHA-VALANA $= \dfrac{\sin L.\, \sin n}{\cos d}$ (B)

See V. 37.

The ÁKSHA-VALANA is called north or south as the point A be north or south to the point D.

And the rectified VALANA D E $=$ D A $+$ A E, when the point A lies between the points D and E, but if the point A be beyond them, the rectified VALANA will be equal to the difference between the ÁKSHA and ÁYANA-VALANA. This also is called north or south as the point E be north or south to the point D.

The ancient astronomers LALLA, S'RÍPATI &c. used the co-versed sin l instead of cos l and the radius for the cos d in (A) and the versed sin n in the place of sin n and radius for the cos d in (B) and hence, the VALANAS, found by them are wrong. BHÁSKARÁCHÁRYA therefore, in order to convince the people of the said mistake made by LALLA, S'RÍPATI, &c. in finding the VALANAS refuted them in several ways in the subsequent parts of this chapter.—B. D.]

30. In either the 1st Libra or the 1st Aries in the equinoctial point of intersection of the equinoctial and ecliptic, the north and south lines of the two circles i. e. their secondaries are different

ÁYANA-VALANA.

and are at a distance* of the extreme declination (of the Sun i. e. 24°) from each other.

31. Hence, the ÁYANA-VALANA will then be equal to the sine of 24° :—The north and south lines of these two circles however are coincident at the solstitial points.

32, 33 and 34. And the north and south lines being there coincident, it follows as a matter of course that the east of those two circles will be the same. Hence at the solstitial points there is no (ÁYANA) VALANA.

When the planet is in any point of the ecliptic between the equinoctial and solstitial points, ÁYANA-VALANA is then found by proportion, or by multiplying the co-sine of the longitude of the planet by the sine of 24°, and dividing the product by the DYUJYÁ or the co-sine of the declination of the planet. This ÁYANA-VALANA is called north or south as the planet be in the ascending or descending signs respectively.

ÁKSHA-VALANA.

Thus in like manner at the point of intersection of the primo vertical and equinoctial, the six o'clock line is the north and south line of the equinoctial, whilst the horizon (of the given place) is the north and south line of the prime vertical. The distance of these north and south lines is equal to the latitude (of the place).

35. Hence at (the east or west point of) the horizon, the ÁKSHA-VALANA is equal to the sine of the latitude. At midday the north and south line of the equinoctial and prime vertical is the same. Hence at midday there is no ÁKSHA-VALANA.

36. For any intervening spot, the ÁKSHA-VALANA is to be found from the sine of the NATA† by proportion.

First, the degrees of NATA are (nearly) to be found by multiplying the time from noon by 90 and dividing the product by the half length of day.

* [By the distance of any two great circles is here meant an arc intercepted between them, of a great circle through the poles of which they pass.—B. D.]

† [Here the NATA is the arc of the prime vertical intercepted between the zenith and the secondary circle to it passing through the place of the planet.—B. D.]

37. Then the sine of the NATA degrees multiplied by the sine of latitude, and divided by the co-sine of the declination of the planet will be the ÁKSHA-VALANA. If the NATA be to the east, the ÁKSHA-VALANA is called north. If west, then it is called south (in the north terrestrial latitude).

SPASHṬA-VALANA.

The sum and difference of the ÁYANA and ÁKSHA-VALANAS must be taken for the SPASHṬA-VALANA, viz. their sum when the ÁYANA and ÁKSHA-VALANAS are both of the same denomination, and their difference when of different denominations i. e. one north and the other south.

38. When the planet is at either the points of the intersection of the ecliptic and prime vertical, the SPASHṬA-VALANA found by adding or subtracting the ÁYANA and ÁKSHA-VALANAS (as they happen to be of the same or different denominations) is for that time at its maximum.

39. But at a point of the ecliptic distant from the point of intersection three signs either forward or backward, there is no SPASHṬA-VALANA: for, at those points the north and the south lines of the two circles are coincident.

40. However, were you to attempt to shew by the use of the versed sine, that there was then no SPASHṬA-VALANA at those points, you could not succeed. The calculation must be worked by the right sine. I repeat this to impress the rule more strongly on your mind.

Another way of refutation of using the versed sine.

41. As all the circles of declination meet at the poles; it is therefore evident that the north and south line perpendicular to the east and west line in the plane of the equinoctial, will fall in the poles.

42. But all the circles of celestial latitude meet in the pole of the ecliptic-called the KADAMBA, 24° distant from the equinoctial pole. And it is this ecliptic pole which causes and makes manifest the VALANA.

43. In the ecliptic poles always lies the north and south

line which is perpendicular to the east and west line in the plane of the ecliptic.

To illustrate this, a circle should be attached to the sphere, taking the equinoctial pole for a centre, and 24° for radius. This circle is called the KADAMBA-BHRAMA-VṚITTA or the circle in which the KADAMBA revolves (round the pole).

The sines in this circle correspond with the sines of the declination.

All the secondary circles to the prime vertical meet in the point of intersection of the meridian and horizon, and this point of intersection is called SAMA i. e. north or south point of horizon.

Now from the planet draw circles on the sphere so as to meet in the SAMA, in the equinoctial pole and also in the ecliptic pole.

The three different kinds of VALANA will now clearly appear between these circles: viz. the ÁKSHA VALANA is the distance between the two circles just described passing through the SAMA and equinoctial pole.

2. The ÁYANA-VALANA is the distance between the circles passing through the ecliptic and equinoctial poles.

3. The SPASHṬA-VALANA is the distance between the circles passing through the SAMA and KADAMBA.

These three VALANAS are at the distance of a quadrant from the planet and are the same in all directions.

Second mode of illustrating the SPASHṬA-VALANA.

48 and 49. Or (to illustrate the subject further) making the planet as the pole of a sphere, draw a circle at 90° from it: then in that circle you will observe the ÁKSHA VALANA—which, in it, is the distance of the point intersected by the equinoctial from the point cut by the prime vertical.

The distance of the point cut by the equinoctial from that cut by the ecliptic is the ÁYANA—and the distance between the points cut by the ecliptic and prime vertical the SPASHṬA-VALANA.

50. In this case the plane of the ecliptic is always east and west—celestial latitude forming its north and south line. Those therefore who (like S'RÍPATI or LALLA) would add the S'ARA celestial latitude to find the VALANA, labour under a grievous delusion.

51. The 1st of Capricorn and the ecliptic pole reach the meridian at the same time (in any latitude): so also with regard to the 1st Cancer. Hence at the solstitial points there is no ÁYANA-VALANA.

52. As the 1st Capricorn revolves in the sphere, so the ecliptic pole revolves in its own small circle (called the KADAMBA-BHRAMA-VRITTA round the pole).

53 and 54. When the 1st of Aquarius or the 1st of Pisces comes to the meridian, the distance in the form of a sine in the KADAMBA-BHRAMA-VRITTA, between the ecliptic pole and the meridian is the ÁYANA-VALANA. This VALANA corresponds with the KRÁNTIJYÁ or the sine of declination found from the degrees corresponding to the time elapsed from the 1st Capricornus leaving the meridian.

55. As the versed sine is like the sagitta and the sine is the half chord (therefore the versed sine of the distance of the ecliptic pole from the meridian will not express the proper quantity of VALANA as has been asserted by LALLA &c.: but the right sine of that distance does so precisely). The ÁYANA-VALANA will be found from the declination of the longitude of the Sun added with three signs or 90°.

56. Those people who have directed that the versed sine of the declination of that point three signs in advance of the Sun should be used, have thereby vitiated the whole calculation. ÁKSHA-VALANA may be in like manner ascertained and illustrated: but it is found by the right sine, (and not by the versed sine).

57. He who prescribes rules at variance with former texts and does not shew the error of their authors is much to be blamed. Hence I am acquitted of blame having thus clearly exposed the errors of my predecessors.

58. The inapplicability of the versed sine may be further illustrated as follows. Make the ecliptic pole the centre and draw the circle called the JINA-VṚITTA with a radius equal to 24°.

Another way of refutation, of using the versed sine.

59. Then make a moveable secondary circle to the ecliptic to revolve on the two ecliptic poles. This circle will pass over the equinoctial poles, when it comes to the end of the sign of Gemini.

60. By whatever number of degrees this secondary circle is advanced beyond the end of Gemini, by precisely the same number of degrees, it is advanced beyond the equinoctial pole, in this small JINA-VṚITTA. The sine of those degrees will be there found to correspond exactly with and increase as does the sine of the declination.

61. And this sine is the ÁYANA-VALANA: This VALANA is the VALANA at the end of the DYNJYA. For the distance between the equinoctial pole and planet is always equal to the arc of which the DYNJYA is the sine i. e. the cosine of the declination.

62. But as the value of the result found is required in terms of the radius, it is consequently to be converted into those terms.

As the JINA-VṚITTA was drawn from the ecliptic pole as centre, with a radius equal to the greatest declination, so now, making the SAMA centre draw a circle round it with a radius equal to the degrees of the place's latitude. (This circle is called AKSHA-VṚITTA.)

63 and 64. To the two SAMAS or north and south points of the horizon as poles, attach a moveable secondary circle to the prime vertical. Now, if this moveable circle be brought over the planet, then its distance counted in the AKSHA-VṚITTA or small circle from the equinoctial pole will be exactly equal to that of the planet from the zenith in the prime vertical. The sine of the planet's zenith distance in the prime vertical, will, when reduced to the value of the radius of ÁKSHA-VṚITTA represent the ÁKSHA-VALANA.

65. As in the AYANA-VALANA so also in this ÁKSHA-VALANA, the result at the end of the DYNJYÁ is found; this therefore must be converted into terms of the radius. From this illustration it is evident that it may be accurately ascertained from the zenith distance in the prime vertical.

66. I will show now how the ÁKSHA-VALANA may be also ascertained from the time from the planets being on the meridian in its diurnal circle. [The rule is as follows.] Add or subtract the S'ANKUTALA [of a given time] to and from the sine of amplitude according as they are of the same or of different denominations (for the BÁHU or BHUJA).

See verse 41, Chap. VII.

67. The sine of the latitude of the given place multiplied by the sine of the ASUS of the time from the planet's being on the meridian, and divided by the square-root of the difference between the squares of the BHUJA (above found) and of the radius, will be exactly the ÁKSHA-VALANA.*

* This rule and the means by which it has been established by BHÁSKARÁCHÁRYA require elucidation.

BHÁSKARA'CHA'RYA first directs that the BA'HU or BHUJA be found for the time of the middle of the eclipse and that a circle parallel to the prime vertical, be drawn having for its centre a point on the axis of the prime vertical distant from the centre of the prime vertical, by the amount of the BA'HU. From this as centre and the KOṬI equal to $=\sqrt{\text{rad}^2-\text{BA'HU}^2}$ as radius draw a circle parallel to the prime vertical. This circle called an UPAVṚITTA will cut the diurnal circle for the time on 2 points equally distant from the meridian. Connect those points by a chord. The half of this chord is the NATAGHAṬI'JYÁ as well in the diurnal circle as in the UPAVRITTA, but as these 2 circles differ in the magnitude, these sines will be the sines of a different number of degrees in each circle. Now the NATAGHAṬI'JYÁ is known, but it is in terms of a large circle. Reduce them to their value in the diurnal circle.

1. If TRIJYÁ : NATAJYÁ : : DYNJYA' : sine of diurnal circle.

This sine in diurnal circle is also sine in UPAVṚITTA.

2. If UPA-VṚITṬA-TRIJYA : this sine : : TRIJYA equal to AKSHAJYÁ.
3. DYNJYA' : this result : : TRIJYA : sine of AKSHA-VALANA

now cancel

and there will remain the rule above stated

$$\frac{\text{NATAJYA} \times \text{AKSHAJYA'}}{\text{UPAVRITTA-TRIJYA'}} = \text{sine of AKSHA-VALANA.}$$

Here our author makes use of the diurnal circle and UPAVṚITTA in term of the equator and prime vertical, whose portions determine the VALANA. The smaller circles being parallel to the larger, the object sought is equally attained. —L. W.

68. Or the ÁKSHA-VALANA may be thus roughly found.

Multiply the time from the planet's being on the meridian and divide the product by the half length of day, the result are the NATA degrees. The sine of these NATA degrees multiplied by the sine of the latitude and divided by the DYNJYÁ or the cosine of the declination, will give the rough ÁKSHA-VALANA.

Further illustration.

69. Place the disc of the Sun at the point at which the diurnal circle intersects the ecliptic. The arc of the disc intercepted between these two circles represents the ÁYANA-VALANA in terms of radius of the disc.

70. This VALANA is equal to the difference between the sine of declination of the centre of the Sun and of the point of intersection of the disc and ecliptic; and it is thus found; multiply the radius of disc by the BHOGYA-KHAṆḌA of the BHUJA of the Sun's longitude and divide by 225.

71. Then multiply this result by sine of 24° and divide by the radius: the quotient is the difference of the two sine of declination. This again multiplied by the radius and divided by the radius of Sun's disc will give the value in terms of the radius (of a great circle).

72. Now in these proportions the radius of the Sun's disc and also radius are in one case multipliers (being in third places), and in the other divisors (being the first terms of the proportion) therefore cancel both. There will then remain rule, multiply the Sun's BHOGYA KHAṆḌA by sine of 24° and divide by 225.

73. And this quantity is equal to the declination of a point of ecliptic 90° in advance of Sun's place. Thus you observe that the VALANA is found by the sine of declination as above alleged, (and not by the versed sine). Abandon therefore, O foolish men, your erroneous rules on this subject.

74. The disc appears declined from the zenith like an umbrella; but the declination is direct to the equinoctial pole:

the proportion of the DYNJYÁ or complement of declination is therefore required to reduce the VALANA found to its proper value in terms of the radius.

End of Chapter VIII. In explanation of the cause of eclipses of the Sun and Moon.

CHAPTER IX.

Called DṚIKKARAMA-VÁSANÁ *on the principles of the Rules for finding the times of the rising and setting of the heavenly bodies.*

Object of the correction called the DṚIKKARMA which is requisite to be applied to the place of the planet, for finding the point of the ecliptic on the horizon when the planet reaches it.

1. A planet is not found on the horizon at the time at which its corresponding point in the ecliptic (or that point of the ecliptic having the same longitude) reaches the horizon, inasmuch as it is elevated above or depressed below the horizon, by the operation of its latitude. A correction called DṚIKKARAMA to find the exact time of rising and setting of a planet, is therefore necessary.

2. When the planet's corresponding point in the ecliptic reaches the horizon, the latitude then does not coincide with the horizon, but with the circle of latitude. The elevation of the latitude above and depression of it below the horizon, is of two sorts, [one of which is caused by the obliquity of the ecliptic and the other by the latitude of the place.] Hence the DṚIKKARAMA is two-fold, i. e. the ÁYANA and the AKSHAJA or ÁKSHA. The detail and mode of performing these two sorts of the correction are now clearly unfolded.

DṚIKKARMA.

3. When the two VALANAS are north and the planet's corresponding point in the ecliptic is in the eastern horizon, the planet is thereby depressed below the horizon by south latitude, and elevated when the planet's latitude is north.

4. When the two kinds of VALANA are south, then the reverse of this takes place; the reverse of this also takes place when the planet's corresponding point is in the western horizon.

[And the difference in the times of rising of the planet and its corresponding point is called the resultant time of the DṚIKKARMA and is found by the following proportions.]

If radius : ÁYANA-VALANA : : what will celestial latitude give?

5. And if cosine of the latitude of the given place : ÁKSHA-VALANA : : what will SPASHṬA S'ARA give?

Multiply the two results thus found by these two proportions, by the radius and divide the products by the DYUJYÁ or cosine of declination.

6 and 7. Take the arcs of these two results (which are sines) and by the ASUS found from the sum of or the difference between these two arcs, the planet is depressed below or elevated above the horizon. The LAGNA or horoscope found by the direct process (as shown in the note on the verse 26, Chapter VII.) when the planet is depressed and by the indirect process (as shown in the same note) when it is elevated, by means of the ASUS above found, is its UDAYA LAGNA rising horoscope or the point of the ecliptic which comes to the eastern horizon at the same time with the planet.

When the planet's corresponding point is in the western horizon, the LAGNA horoscope found then by the rule converse of that above given, by means of the place of the planet added with 6 signs, is its ASTA LAGNA setting horoscope or the point of the ecliptic which is on the eastern horizon when the planet comes to the western horizon.

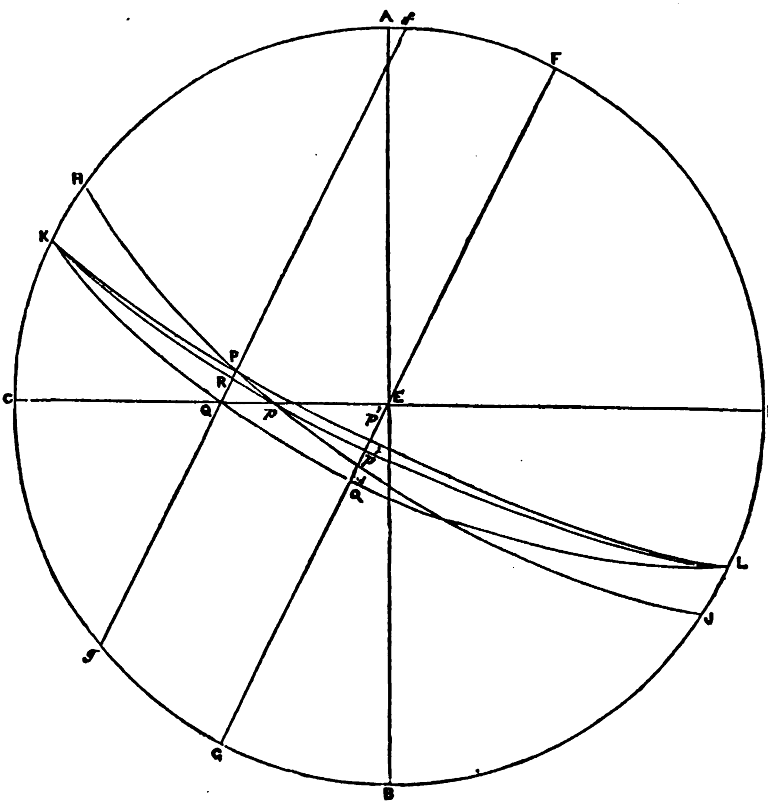
8 and 9. For the fixed stars whose latitudes are very considerable the resulted time of the DṚIKKARMA is found in a

different way. Find the ascensional difference from the mean declination of the star, i. e. from the declination of its corresponding point in the ecliptic, and also from that applied with the latitude, i. e. from the true declination. The ASUS found from the sum of or the difference between the ascensional differences just found, as the mean and true declinations are of the different or of the same denominations respectively, are the ASUS of depression or elevation depending on the AKSHA DṚIKKARMA. (Find also the time depending on the ÁYANA-DṚIKKARMA) : and from the sum of or the difference between them, as they may be of the same or different denominations, the UDAYA LAGNA or ASTA LAGNA may be ascertained as above found (in the 6th and 7th verses).*

* Let A D B C be the meridian; C E D the horizon, A the zenith; E the east point of the horizon; F E G the equinoctial; K the north pole; L the south; P the planet; p its corresponding point in the ecliptic; H P p J the secondary to the ecliptic passing through the planet P, and hence p P the latitude. Let f P g the diurnal circle passing through the planet P and hence p R the rectified latitude.

Now, when the corresponding place of the planet is in the horizon, it is then evident from the accompanying figure, that the planet is elevated above or depressed below the horizon by its latitude p P and as it is very difficult to find the elevation or depression at once, it is therefore ascertained by means of its two parts, the one of which is from the horizon to the circle of declination, i. e. Q to R. This partial elevation or depression takes place by the planet's rectified latitude p R. And the other part of the elevation or depression is from the circle of declination to the circle of latitude; i. e. from R to P and this occurs by the planet's mean latitude p P. From the sum or difference of these two parts, the exact elevation of the planet above the horizon or the depression below it, can be determined. When the terrestrial latitude, of the given place is north and the planet's corresponding place in the ecliptic is in the eastern horizon, the A'KSHA-VALANA is then north and the circle of declination is elevated above the horizon to the north. For this reason, when the A'KSHA-VALANA is north, the planet will be elevated above the eastern horizon if its latitude be north, and if it be south, the planet will be depressed below the horizon. But the reverse of this takes place when the A'KSHA-VALANA is south which occurs on account of the south latitude of the given place, i. e. when the A'KSHA-VALANA is south, the circle of declination is depressed below the horizon to the north and hence the planet is depressed below it, if its latitude be north, and if it be south, the planet is elevated above the horizon.

Again, when the planet's longitude terminates in the six ascending signs, it is evident that the ÁYANA-VALANA becomes then north, and the north pole of the ecliptic is elevated above the circle of declination passing through the planet. Hence, when the A'YANA-VALANA is north, the planet is elevated above or depressed below the circle of declination by its mean latitude, as it is north or south. But the reverse of this takes place, when the A'YANA-VALANA is south, i. e. the planet is depressed below or elevated above the circle of declination, as its latitude is north or south. Because when the A'YANA-VALANA is south



the north pole of the ecliptic lies below the circle of declination and the south above it.

Again, when the planet is in the western horizon, the circle of declination passing through the place of the planet in the ecliptic lies to the north above the horizon, but the AKSHA-VALANA, becomes south and hence the reverse takes place of what is said about the elevation or depression when the planet is in the eastern horizon. But as to the ÁYANA-VALANA, it becomes north when the longitude of the planet terminates in the ascending six signs and the north pole of the ecliptic lies below the circle of declination. Hence the depression of the planet takes place when its latitude is north and the elevation when the latitude is south. But when the longitude of the planet terminates in the discending six signs, the ÁYANA-VALANA becomes then south and the north pole of the ecliptic lies above the circle of declination. For this reason, the elevation of the planet takes place when its latitude is north, and the depression when it is south. Thus in the western horizon the elevations and depressions of the planet are opposite to those when the planet is in the eastern horizon.

Now, the time elapsed from the planet's rising when it is elevated above the horizon and the time which the planet will take to rise when it is depressed below the horizon, are found in the following manner.

10. The [ASPASHṬA] S'ARA or true latitude [of the planet] multiplied by the DYUJYA' or cosine of declination of the point of the ecliptic, three signs in advance of the planet's corresponding point and di-

To find the value of celestial latitude in terms of a circle of declination, to render it fit to be added to or subtracted from declination.

See the figure above described in which the angle Q K R or the equinoctial arc $Q' p'$ denotes the time of elevation of the planet from Q to R, and the time of elevation of the planet from R to P is denoted either by the angle P K R or by the equinoctial arc $P' p'$. Out of these two times $Q' p'$ and $P' p'$, we show at first how to find $P' p'$.

In the triangle $P p R$, $P p =$ the latitude of the planet, $\angle P p R =$ the A'YANA-VALANA and $\angle P R p = ⅃$, and

$\therefore R : \sin P p R = \sin P p : \sin R P$;

or if radius

: sin of A'YANA-VALANA

= the sine of latitude

: sin R P.

Again, by the similar triangles K P R and K P' p'

$\sin K P : \sin R P = \sin K P' : \sin P' p'$,

here, $\sin K P =$ cosine of declination and $K P' = R$,

$$\therefore \sin P' p' = \frac{R \times \sin R P}{\text{cos of declination}}$$

Now, the time $p' Q'$ is found as follows.

In the triangle $p R Q$, $p R =$ the SPASHṬA-S'ARA which can be found by the rule given in the V. 10 of this chapter, $\angle R p Q =$ AKSHA-VALANA and $\angle R Q p =$ co-latitude of place nearly

and $\therefore \sin p Q R : \sin R p Q :: \sin p R : \sin R Q$

or, if cosine of latitude,

: sine of AKSHA-VALANA,

= SPASHṬA-S'ARA

: sin R Q;

again, by the triangles K Q R, K Q' p',

$\sin K Q : \sin Q R = \sin K Q' : \sin p' Q'$;

here, $\sin K Q =$ cosine of declination and sine $K Q' = R$,

$$\therefore \sin p' Q' = \frac{R \times \sin Q R}{\text{cos of declination.}}$$

If both of these times thus found, be of the elevation or both of the depression, the planet will be elevated above or depressed below the horizon in the time equal to their sum, and if one of these be that which the planet takes for its elevation and the other for its depression, the planet will be elevated above or depressed below the horizon in the time equal to their difference as the remainder is of the time of elevation or of that of the depression. The sum or difference of the two times just found is called the resulted time of the DṚIK-KARMA in the S'IDDHANTAS.

That point of the ecliptic which is on the eastern horizon when the planet reaches it, is called the UDAYA LAGNA rising horoscope of the planet. As it is necessary to know this UDAYA LAGNA for finding the time of the planet's rising, we are now going to show how to find the rising horoscope. If the planet is depressed by the resulted time above mentioned, it is evident that when the planet will come to the eastern horizon, its corresponding place in the

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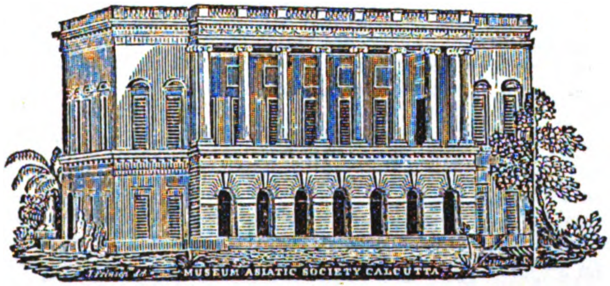

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vided by the radius becomes [nearly] the SPASHTA or rectified latitude, [i. e. the arc of the circle of declination intercepted between the planet's corresponding point in the ecliptic and the diurnal circle passing through the planet]. This rectified latitude is used when it is to be applied to the mean declination and also in the ÁKSHA DṚIKKARMA.*

11. The celestial latitude is not reduced by BRAHMAGUPTA

ecliptic will be elevated above it by the resulted time. For this reason, having assumed the corresponding place of the planet for the Sun, find the horoscope by the direct process through the resulted time and this will be the rising horoscope. But if the planet be elevated above the horizon by the resulted time, its corresponding place will then be depressed below it by the same time when the planet will come to it. Therefore, the horoscope found by the indirect process through the resulted time; will be the rising horoscope of the planet.

That point of the ecliptic which is on the eastern horizon when the planet comes to the western horizon, is called the ASTA LAGNA or setting horoscope of the planet. As it is requisite to know the setting horoscope for finding the time of setting of the planet, we therefore now show the way for finding the setting horoscope. If the planet be depressed below the western horizon by the resulted time, it is plane that when the planet will reaches it, its corresponding place will be elevated above it by the resulted time and consequently the corresponding place of the planet added with six signs will be depressed below the eastern horizon by the same time. Therefore, assume the corresponding place of the planet added with six signs for the Sun and find the horoscope by the indirect process, through the resulted time and this will be the ASTA LAGNA setting horoscope. But if the planet be depressed below the western horizon, its corresponding place added with six signs will then be elevated above the eastern horizon by the resulted time and hence the horoscope found by the direct process will then be the ASTA LAGNA setting horoscope.

Now the time p' Q' which is determined above through the triangle p R Q, is not the exact one, because, in that triangle the angle p Q R is assumed equal to the co-latitude of the given place, but it cannot be exactly equal to that, and consequently the time p' Q' thus determined cannot be the exact time. But no considerable error is caused in the time p' Q' thus found, if the latitude be of a planet, as it is always small. As to the star whose latitude is considerable, the time p' Q' thus found cannot be the exact time. The exact time can be found as follows.

See the preceding figure and in that take R for a star and p the intersecting point of the ecliptic, and the circle of declination passing through the star R then p p' is called the mean declination of the star, R p, the rectified latitude and R p' the rectified declination.

Now, find the ascensional difference E p' through the mean declination p p' and the ascensional difference E Q' through the rectified declination R p' or Q Q'. Find the difference between these two ascensional differences and this difference will be equal to p' Q' i. e. E Q'—E p' = p' Q'. But it occurs then when p and R are in the same side of the equinoctial F G and when p is in one side and R in the other of the equinoctial, it is evident that p' Q' in this case will be equal to the sum of the two ascensional differences.—B. D.]

* This rule is admitted by BHÁSKARÁCHÁRYA to be incorrect; but the error being small, is neglected. Instead of using the DYUJYÁ, the YASUṬI should have been adopted.

Omission of the last mentioned correction or reduction of Celestial latitude to its value in declination, by BRAHMAGUPTA and others.

and other early astronomers to its value in declination: and the reason of this omission, seems to have been its smallness of amount. And also it is the uncorrected latitude which is used in finding the half duration of the eclipses and in their projections &c.

12. As the constellations are fixed, their latitudes as given in the books of these early astronomers are the SPASHAṬ-S'ARAS, i. e. the reduced values of the latitudes so as to render them fit to be added to or subtracted from the declination; and the DHRUVAS or longitude of these constellations are given, after being corrected by the ÁYANA DṚIKKARMA so as to suit those corrected latitudes that is, the star will appear to rise at the equator at the same time with longitude found by the correction.

Let *a d* be equinoctial and P the equinoctial pole,

d b = Ecliptic,

b s = Celestial latitude,

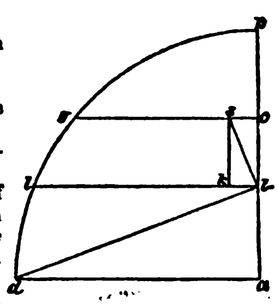
b c = Celestial latitude reduced to its value in declination is KOṬI,

s c = BHUJA being arc of diurnal circle *c s g*

s c = *k b* portion of diurnal circle of the planet's longitude at *b*.

The triangle *s c b* or *s k b* is assumed to be a DIG-VALANAJÁ TRYASRA.

The angle *s b c* = ÁYANA-VALANA or the angle of the inclination of *s b* which goes to ecliptic pole with *b c* which goes to equinoctial pole.



Hence this triangle *s b c* is called DIG-VALANAJÁ TRYSRA, the angle *s b c* varying with the ÁYANA-VALANA. If *b* were at the 1st Cancer, then the north line *a b c* which goes to the pole would go also to the ecliptic pole.

Hence the ASPASHṬA SA'RA, and SPASHṬA S'ARA of a star of 90° of latitude being both represented by *b c* would be the same. To the longitude of a star being 270°, its ASPASHṬA and SPASHṬA SÁRA would be the same.—L. W.

[The rule stated in this verse is founded upon the following principle.

Assuming the triangle *s b c* as a plane right-angled triangle and the angle *s b c*, as the declination of the point of the ecliptic three signs in advance of the planet's corresponding place, because this declination is nearly equal to the ÁYANA-VALANA, we have,

$$\sin s\,c\,b : \cos s\,b\,c = b\,s : b\,c;$$

or R : YASHTI or nearly the cosine of the declination of the planet's place 9°0 + = Celestial latitude : rectified latitude.—B. D.]

13. Those astronomers, who have mentioned that celestial latitude is an arc of a circle of declination, are stupid. Were the celestial latitude nothing more than an arc of a circle of declination, then why should they or others have ever had recourse to the ÁYANA DṚIKKARMA at all? (The planets or stars would appear on the six o'clock line at the time that the corresponding degree of the ecliptic appeared there.)

BHA'SKARA'CHÁRYA exposes the incorrect theory of certain of his predecessors, by quoting their own practice which is irreconcilable with their own theory.

14. How moreover have these same astronomers in delineating an eclipse marked off the Moon's latitude in the middle of the eclipse on SPASHṬA-VALANA-SÚTRA or on the line denoting the secondary circle to the ecliptic? and how also have they drawn perpendicularly on the VALANA-SÚTRA or the line representing the ecliptic, the latitudes of the Moon at the commencement and termination of the eclipse.

15. How moreover, have they made the latitude KOṬI, i. e. perpendicular to the ecliptic and thus found the half duration of the eclipse? If the latitude were of this nature, it would never be ascertained by the proportion (which is used in finding it).

16. A certain astronomer has (first) erroneously stated the DṚIKKARMA and VALANA by the versed sine. This course has been followed by others who followed him like blind men following each other in succession: [without seeing their way].

Censure of the astronomers who erroneously used the versed sine in the DṚIKKARMA and VALANA.

17. BRAHMAGUPTA's rule, however, is wholly unexceptionable, but it has been misinterpreted by his followers. My observations cannot be said to be presumptuous, but if they are alleged to be so, I have only to request able mathematicians to weigh them with candour.

Praise of BRAHMAGUPTA.

18. The DṚIKKARMA and VALANA found by the former astro-

nomers through the versed sine are erroneous: And I shall now give an instance in proof of their error.

An instance in proof of the error.

19 and 20. In any place having latitude less than 24° N. multiply the sine of the latitude of the place by the radius and divide the product by the sine of 24° or the sine of the obliquity of the ecliptic and take the arc in degrees of the result found. And find the point of the ecliptic, the degrees just found in advance of the 1st Aries. Now, if from this point the planet's corresponding point on the ecliptic three signs backwards or forwards, be on the western or eastern horizon respectively, then the ecliptic will coincide with the vertical circle, and the horizon will consequently be secondary to the ecliptic. Hence the planet will not quit the horizon, though it be at a distance, of extreme latitude from its corresponding point in the ecliptic [which is on the horizon], as the celestial latitude is perpendicular to the ecliptic.*

21. In this case the resulted times of the DṚIKKARMA being of exactly the same amount but one being *plus* and the other *minus*, neutralize each other [and hence there is no correction]. Now this result would not be obtained by using the versed sine—hence let the right sine (as prescribed) be always used for the DṚIKKARMA.

* [It is evident that the longitude of this point is equal to the arc through which it is found, and as the point of the ecliptic 3 signs backwards or forwards from this point is assumed on the horizon, this point therefore will at that time be the nonagesimal, and as the longitude of that point or nonagesimal is less than 90° the declination of this point will be north. This declination equals to the latitude in question. For

$\because$ The sine of the latitude of the point $= \dfrac{\text{R} \times \text{sin latitude}}{\text{sin } 24^\circ}$ (by the assumption)

$\therefore$ sin latitude $= \dfrac{\text{sin } 24^\circ \times \text{sin longitude of the point}}{\text{Radius}}$, but this = sin declination.

$\therefore$ The declination of that point or nonagesimal equal to the latitude of the place. And hence, if the latitude be north the nonagesimal will be in the zenith. For this reason the ecliptic will coincide with the vertical circle.—B. D.]

22. Again here, in like manner, it is from the two VALANAS having different denominations, but equal values, that they mutually destroy each other. By using the versed sine, they would not have equal amounts, hence the VALANAS must be found by the right sine.

[In illustration of the fact that the VALANA does not correspond with the versed sine, but the right sine BHÁSKARÁCHÁRYA gives as an example.]

23. When the Sun comes to the zenith [of the place where the latitude is less than 24°], and consequently the ecliptic coincides with the vertical circle, the SPASHṬA VALANA then evidently appears to be equal to the sine of the amplitude of the ecliptic point 90° in advance of the Sun's place in the horizon. If you, my friend, expert in spherics, can make the SPASHṬA VALANA equal to the sine of amplitude by means of the versed sine, then I will hold the VALANA found in the DHÍVṚIDDHIDA TANTRA by LALLA and in the other works to be correct.

[To this BHÁSKARÁCHÁRYA adds a further most important and curious illustration :]

24. In the place where the latitude is 66° N. when the Sun at the time of his rising is in 1st Aries, 1st Taurus, 1st Pisces, or in 1st Aquarius, he will then be eclipsed in his southern limb, because the ecliptic then coincides with the horizon. Therefore, tell me how the SPASHṬA VALANA will be equal to the radius by means of the versed sine!

[In the same manner the DṚIKKARMA calculation as it depends on the VALANA, must be made by the right sine and not by the versed sine and for the same reasons.]

Cause of error in LALLA and others, stated.

25. Even clever men are frequently led astray by conceit in their own quick intelligence, by their too hasty zeal and anxiety for distinction, by their confidence in others and by their own negligence or inadvertence, when it is thus with the wise, what need I say of fool? others, however, have said :—

26. Those given to the service of courtezans and bad poets,

are both distinguished by their disregard of the criticisms and reflections of the world, by their breach of the rules of time and metres, and their destruction of their substance and of their subject, being beguiled by the vain delight they feel towards the object of their taste.

End of Chapter IX. called DRIKKARMA-VÁSANÁ.

CHAPTER X.

Called S'RINGONNATI-VÁSANÁ *in explanation of the cause of the Phases of the Moon.*

1. This ball of nectar the Moon being in contact with rays of the Sun, is always illuminated by her shinings on that side turned towards the Sun. The side opposite to the Sun dark as the raven black locks of a young damsel, is obscured by being in its own shadow, just as that half of a water-pot which is turned from the Sun, is obscured by its own shadow.

2. At the conjunction, the Moon is between us and the Sun: and its lower half which is then visible to the inhabitants of the earth, being turned from the Sun is obscured in darkness.

That half again of the Moon when it has moved to the distance of six signs from the Sun, appears to us at the period of full Moon brilliant with light.

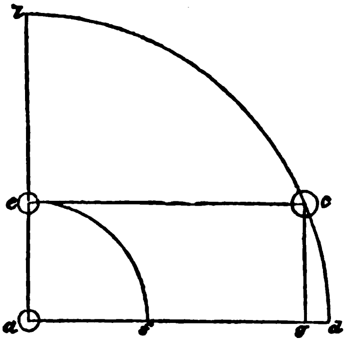
3. Draw a line from the earth to the Sun's orbit at a distance of 90° from the Moon, and find also a point in the Sun's orbit (in the direction where the Moon is) at a distance equal to that of the Moon from the earth. When the Sun reaches the point just found, he comes in the line perpendicular at the Moon to that drawn from the earth to the Moon. Then the Sun illumines half of the visible side of the

Moon. That is when the Moon is 85° . . 45′ from the Sun east or west, it will appear half full to us.*

4. The illuminated portion of the Moon gradually increases as it recedes from the Sun: and the dark portion increases as it approaches the Sun. As this sea-born globe of water (the Moon) is a sphere, its horns assume a pointed or cusped appearance (varying in acuteness according to its distance from the Sun).

5. (To illustrate the subject, a diagram should be drawn as follows). Let the distance north and south between the Sun and Moon represent the BHUJA, the upright distance between them the KOṬI and the line joining their centres the hypothenuse. The Sun is in the origin of the BHUJA which stretches in the direction where the Moon is, the line perpendicular at the end of the BHUJA is KOṬI at the extremity of which is the Moon and the line stretching (from the Moon) in the direction of the Sun is the hypothenuse. The Sun gives light (to the Moon) through the direction of the hypothenuse.

Diagram for illustrating the subject.



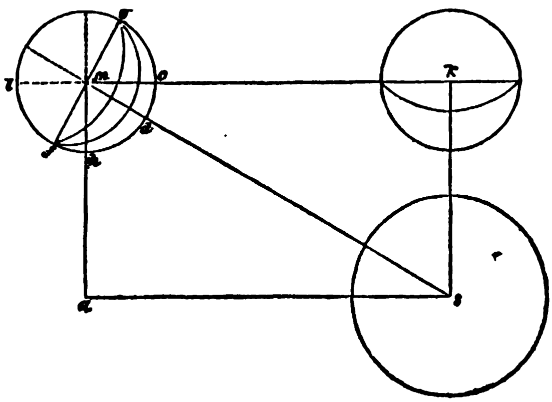
* This is thus illustrated. Let *a* represent the Earth, *b c d* the orbit of the Sun, *e f* = do. of the Moon. Then it is obvious that half of the side of the Moon visible to us will be illuminated when the Sun is at *c* and not at *d*, when the Sun is at *d* it will illumine more than half of the Moon's disc; *b c* is less than a quadrant by the arc *c d*, the sine of which *a e* or *e g* in terms of the radius of the Sun's orbit, equals to the Moon's distance from the earth. L. W.

[The arc *b c* can be found as follows:— In the triangle *a e c* right angled at *e*, *a e* = 51566 YOJANAS, *a c* = 689377 YOJANAS according to the SIDDHA'NTAS.

$$\text{Then, } \cos e\,a\,c = \frac{a\,e}{a\,c} = \frac{51566}{689377} = .0748 = \cos 85^\circ \,..\, 43'$$

$$\therefore \text{ arc } b\,c = 85^\circ \,..\, 45' \text{ nearly.— B. D.]}$$

[For instance



Let S be the Sun and *m* the Moon, then *a* S = BHUJA, *a m* = KOṬI, *m* S = hypothenuse. Then *f g* a line drawn at right angles to extremity of hypotenuse will represent line of direction of the enlightened horns and the angle *h m d* opposite to BHUJA will be equal to $\angle$ *g m c* = the amount of angle by which the northern cusp is elevated and southern depressed,—were the Moon at *k*, there would be no elevation of either cusp either way. For the hypothenuse will also bisect the white part of the Moon. If the Sun is north of the Moon, the north cusp of the Moon is elevated: if south the southern cusp. L. W.]

[Mr. Wilkinson has extracted the following two verses from the GAṆITÁDHYÁYA.

I. When the latitude is 66° N. and the Sun is rising in 1st Aries, then the ecliptic will coincide with the horizon; now suppose the Moon to be in 1st Capricorn, then it will appear to be bisected by the meridian and the eastern half will be enlightened.

But according to BRAHMAGUPTA this would not occur, for he has declared that the KOṬI will be equal to radius in this case whereas it is obviously "nil," and it is the BHUJA which is equal to radius when there is no north and south difference between the Sun and Moon then the KOṬI would be equal to the hypothenuse or radius and the BHUJA would be "nil."

II.* And the Moon's horns are of equal altitude when there is no BHUJA, whilst they become perpendicular when there is no KOṬI. That the KOṬI and BHUJA shall at one and the same time be equal to radius is an obvious incompatibility. But what business have I with dwelling on the exposure of these errors? BRAHMAGUPTA has here shown wisdom indeed, and I offer him my reverent submission !]

6. I have thus only briefly treated of the principles of the subjects mentioned in the Chapters on MADHYAGATI &c. fearing to lengthen my work; but the talented astronomer should understand the principles of all the subjects in completion, because this is the result to be obtained by a complete knowledge of the spheric.

End of Chapter X. called S'RINGONNATI-VÁSANÁ.

CHAPTER XI.

Called YANTRÁDHYÁYA, *on the use of astronomical instruments.*

1. As minute portions of time elapsed from sun-rise cannot be ascertained without instruments, I shall therefore briefly detail a few instruments which are of established use for this purpose.

Object of the Chapter.

2. The Armillary sphere, NÁDÍ-VALAYA (the equinoctial), the YASHṬI or staff, the gnomon, the GHAṬI or clepsydra, the circle, the semi-circle, the quadrant, and the PHALAKA : but of all instruments, it is "INGENUITY" which is the best.

* BHÁSKARÁCHÁRYA is here very severe on BRAHMAGUPTA who of all his predecessors is evidently his favorite, but truth seemed to require this condemnation. He at the same here does justice to ÁRYA-BHATTA and the author of the SÚRYA-SIDDHA'NTA. They both justly concur in saying there is no KOṬI in this case.—L. W.

3 and 4. (This instrument is to be made as before described, placing the BHAGOLA starry sphere, which consists of the ecliptic, diurnal circles, the Moon's path, and the circles of declination &c. within the KHAGOLA celestial sphere, which consists of the horizon, meridian, prime vertical, six o'clock line, and other circles which remain fixed in a given latitude). Bring the place of the Sun on the ecliptic to the eastern horizon: and mark the point of the equinoctial (in the BHAGOLA) intersected by the horizon, viz. east point. Having made the horizon as level as water, turn the BHAGOLA westward till the Sun throws its shadow on the centre of the Earth. The distance between the mark made on the equinoctial and the now eastern point of the horizon will represent the time from sun-rise.

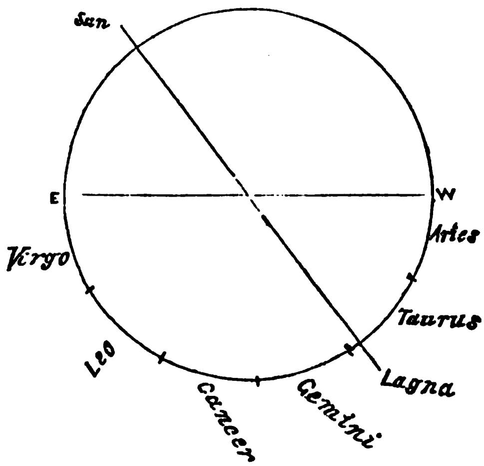
Use of Armillary Sphere.

5 and 6. The LAGNA or horoscope will then be found in that point of the ecliptic which is cut by the horizon.

The NA'DI-VALAYA.

Take a wooden circle and divide its outer rim into 60 GHAṬIKÁS: Then place the twelve signs of the ecliptic on both sides, but instead of making each sign of equal extent, they must be made each with such variable arcs as shall correspond with their periods of rising in the place of observation (the twelve periods are to be thus marked on either side, which are to be again each subdivided into two HORÁS (or hours), three DRESHKÁṆAS, into NAVÁNS'AS or ninths of 3° . . 20′ each, twelfths of 2° . . 10′ and into TRINS'ÁNS'ÁS or thirtieths. These are called the SHAḌVARGA or six classes). These signs, however, must be inscribed in the inverse order of the signs, that is 1st Aries, then Taurus to the west or right of Aries and so on. Then place this circle on the polar axis of the KHAGOLA at the centre of the Earth (the polar axis should be elevated to the height of the pole).

Now find the Sun's longitude in signs, degrees, &c. for the sun-rise of the given day (by calculation) and find the same degree in the circle. Mark there the Sun's place, turn the circle round the axis, so that the shadow of the axis will fall on the mark of the Sun's place at sun-rise and then fix the circle. Now as the Sun rises, the shadow of the axis will advance from the mark made for the point of sun-rise to the nadir and will indicate the hour from sun-rise, and also the LAGNA (horoscope) : the number of hours will be seen between the point of sun-rise and the shadow : and the LAGNA will be found on the shadow itself. [While the Sun goes from east to west the



shadow travels from west to east and hence the signs with their periods of rising must be reversed in order—the arc from W to LAGNA represents the hour arc : and the LAGNA is at the word LAGNA in the accompanying figure.—L. W.]

7. Or, if this circle marked as above, be placed on any axis elevated to the altitude of the pole, then the distance from the shadow of the axis to the lowest part of the circle will represent the time to or from midday.

8. A GHAṬI made of copper like the lower half of a water-pot, should have a large hole bored in its bottom. See how often it is filled and falls to the bottom of the pail of water on which it is placed. Divide 60 GHAṬIS of day and night by the quotient

The GHAṬI or clepsydra.

and it will give the measure of the clepsydra. (If it is filled 60 times, then the GHAṬI will be of one GHAṬIKÁ; if 24 times it will be of one hour or 2½ GHAṬIKÁS.)

Gnomon.

9. For a gnomon take a cylindrical piece of ivory, and let it be turned on a lathe, taking care that the circumference be equal above and below. From its shadow may be ascertained the points of the compass, the place of observer, including latitude &c. and times (as has been elsewhere explained).

The CHAKRA or circle.

10. The circle should be marked with 360° on its outer circumference, and should be suspended by a string or chain moveable on the circumference. The horizon or Earth is supposed to be at the distance of three signs or 90° from the point at which it is suspended: the point opposite to that point being the zenith.

11. Through its centre put a thin axis: and placing the circle in a vertical plane, so as to catch the shadow of the Sun: the degrees passed over by the axis from the place denominated the Earth, will be altitude:

12. And the arc to the point denominated the zenith, will be that of the zenith distance.

Some former astronomers have given the following rule for making a rough calculation of the time, viz. multiply the half length of day by the obtained altitude and divide the product by the meridian altitude, the quotient will be the time sought.

To find the longitudes of planets by the circle.

13. First let the circle be so held or fixed that any two of the following fixed stars appear to touch the circumference, viz. MAGHÁ (α Leonis, Regulus), PUSHYA (δ Cancri), REVATÍ (ζ Piscium) and S'ATATÁRAKÁ (or λ Aquarii). [These stars are on the ecliptic and having no latitude, are to be preferred.] Or, that any star (out of the CHITRÁ or α Virginis Spica &c.) having very inconsiderable latitude, and the planet whose longitude is required and which is at a considerable distance from the star, appear to touch the circumference.

14 and 15. Then look from the bottom of the circle along its plane, so that the planet appear opposite the axis; and still holding it on the plane of the ecliptic, observe also any of the above mentioned stars. The observed distance between the planet and the star, if added to the star's longitude, when the star is west, and subtracted when east of the planet, will give the planet's longitude.

Semi-circle and quadrant. The half of a circle is called a CHÁPA or semicircle. The half of a semicircle is called TURÍYA or a quadrant.

PHALAKA-YANTRA. 16. As others have not ascertained happily the apparent time by observations of altitudes in a vertical circle, I have therefore laboured myself in devising an instrument called PHALAKA YANTRA, the uses of which I now proceed to explain perspicuously. It contains in itself the essence of all our calculations which are founded on the true principles of the Doctrine of the Sphere.

Addresses to the Sun. 17. I BHÁSKARA now proceed to describe this excellent instrument, which is calculated to remove always the darkness of ignorance, which is moreover the delight of clever astronomers and is founded on the shadow of its axis: it is also eminently serviceable in ascertaining the time, and in illustrating truths of astronomy, and therefore valued by the professors of that science. It is distinguished by having a circle in its centre. I proceed to describe this instrument after invoking that bright God of day, the Sun, which is distinguished by the epithets I have above given to the instrument viz. he is eternal and removes obscurity and cold: he makes the lotus to flower and is ever shining: he easily points out the time of the day and season and year, and makes the planets and stars to shine. He is worthy of worship from the virtuous and resides in the centre of his orb.*

* This verse is another instance of the double entendre, in which even the

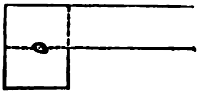
18. Let a clever astronomer make a PHALAKA or board of a plane rectangular and quadrilateral form, the height being 90 digits, and the breadth 180 digits. Let him halve its breadth and at the point thus found, attach a moveable chain by which to hold it: from that point of suspension let him draw a perpendicular which is called the LAMBA-REKHÁ.

19. Let him divide this perpendicular into 90 equal parts which will be also digits, and through them draw lines parallel to the top and bottom to the edges: these are called sines.

20. At that point of the perpendicular intersected by the 30th sine at the 30th digit, a small hole is to be bored, and in it is to be placed a pin of any length which is to be considered as the axis.

21. From this hole as centre draw a circle (with a radius of 30 digits: the circle will then cut the 60th sine), 60 digits forming the diameter. Now mark the circumference of this circle with 60 GHAṬIS and 360 degrees, each degree being subdivided into 10 PALAS.

22. Let a thin PAṬṬIKÁ or index arm with a hole at one end be made of the length of 60 digits and let it be so marked. [The breadth of the end where the hole is bored should be of one digit whilst the breadth of the whole PAṬṬIKÁ be of half digit. Let the PAṬṬIKÁ be so suspended by the pin above mentioned, that one side may coincide with the LAMBA-REKHÁ. The accompanying figure will represent the form of the PAṬṬIKÁ.



The rough ascensional difference in PALAS determined by the KHAṆḌAKAS or parts, being divided by 19, will here become the sine of the ascensional difference (adapted to this instrument.*)

best authors occasionally indulge. All the epithets given to the instrument apply in the original also to the Sun. This kind of double meaning of course does not admit of translation.—L. W.

* The sines of ascensional difference for each sign of the ecliptic were found by the following proportions.

23. The numbers 4, 11, 17, 18, 13, 5 multiplied severally by the AKSHA-KARṆA and divided by 12, will be the KHANḌAKAS or portions at the given place; each of these being for each 15 degrees (of BHUJA of the Sun's longitude) respectively.

24. Now find the Sun's true longitude by applying the precession of the equinoxes to the Sun's place, and adding together as many portions as correspond to the BHUJA of the Sun's longitude above found, divide by 60 and add the quotient to AKSHA-KARNA. Now multiply the result by 10 and divide by 4 (or multiply by 2½). The quotient is here called the YASHṬI in digits and the number of digits thus found is to be marked off on the arm of the PAṬṬIKÁ counting from its hole penetrated by the axis.

25. Now hold the instrument so that the rays of the Sun shall illuminate both of its sides (to secure its being in a vertical circle): the place in the circumference marked out by the shadow of the axis is assumed to be the Sun's place.

26. Now place the index arm on the axis and putting it over the Sun's place, from the point at the end of the YASHṬI set off carefully above or below (parallel to the LAMBA-REKHÁ) on the instrument, the sine of the ascensional difference above found, setting it off above if the Sun be in the northern

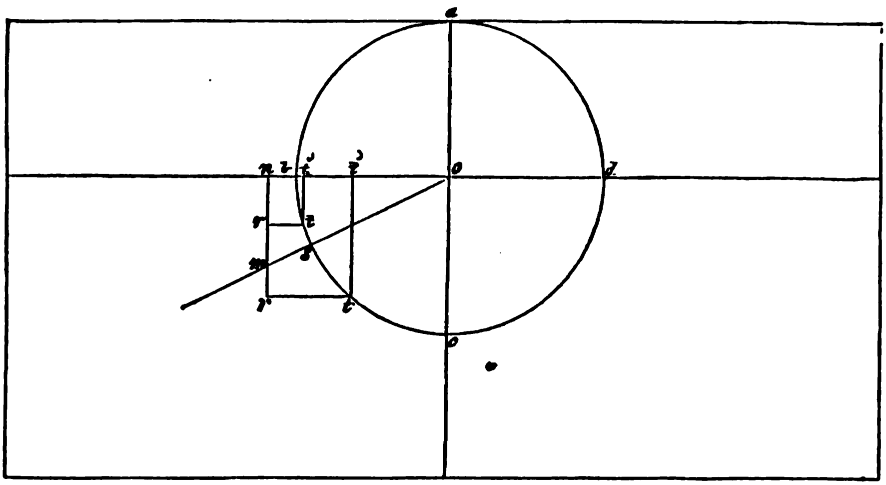
1. If cosine of latitude : sine of lat. } : : what will sine of declination of 1 sign or 2 or 3 signs, give.
or as 12 : PALABHA' } : KUJYA' of 1, 2 or 3 signs.

2. If cosine of declination : this result : : what will radius : sine of ascensional difference in KALÁS.

The arc of this will give ascensional difference. This is the plain rule: but BHA'SKARA'CHA'RYA had recourse to another short rule by which the ascensional differences for 1, 2 and 3 signs, for the place in which the PALABHA' was 1 digit, were 10, 8, 3⅓ PALAS. These three multiplied by PALABHA' would give the ascensional differences with tolerable accuracy for a place of any latitude not having a greater PALABHA' than 8 digits. Now take these three PALÁTMAKAS 10, 8, 3⅓ and multiplied by six, then the PALAS of time will be reduced to ASUS. These are found with a radius of 3438: to reduce them to the value of a radius of 30 digits say,

As $3438 : 10 \times 6 = 60' : : 30 \text{ digits} : \dfrac{60 \times 30}{3438}$ = quantity of CHARA for 1 sign in this instrument, but instead of multiplying the 10 by 6 × 30 or 180 and dividing by 3438, the author taking $180 = \frac{1}{19}$ part of 3438, divides at once by 19.—L. W.

hemisphere, and below if it be in the southern hemisphere. The distance from the point where the sine which meeting the end of the sine of the ascensional difference thus set off, cuts the circle, to the lowest part of the circle will represent the GHAṬIS to or after midday.*



* In the accompanying diagram of the PHALAKA YANTRA, *o* is the centre of the circle *a b c* and the line *o n* passing through *o* is called MADHYAJYA' or middle sine. If the shadow of the pin touches the circumference in S when the instrument is held in the vertical circle passing through the Sun, S *b* will then be the zenith distance of the Sun. From this the time to or after midday can be found in the following manner.

Let a = altitude of the Sun,
d = declination,
A = ascensional difference,
l = north latitude of the place,
p = degrees in time to or after midday.

Then, we have the equation which is common in the astronomical works,

$$\cos p = \frac{R^2 . \sin a \mp R . \sin l . \sin d}{\cos l . \cos d};$$

$$= \frac{R^2 . \sin a}{\cos l . \cos d} \mp \frac{\tan l . \tan d}{R};$$

here, when the latitude is north, the second term becomes minus or plus as the declination is north or south respectively.

But $\dfrac{\tan l . \tan d}{R}$ = sin A or sine of ascensional difference.

$$\therefore \quad \cos p = \frac{R^2 . \sin a}{\cos l . \cos d} \mp \sin . A.$$

27. Set off the time from midday on the instrument counting from the LAMBA-REKHÁ; from the end of the sine of this time, set off the sine of ascensional difference in a line parallel to the

To find the place of the shadow of axis from time.

Now, $\cos l : R = 12 : h$ i. e. AKSHAKARṆA (See Chapter VII. v. 45.)

$$\text{or } \frac{R}{\cos l} = \frac{h}{12},$$

$$\therefore\ \cos p = \frac{h}{12} \cdot \frac{R \cdot \sin a}{\cos d} \mp \sin A$$

$$= y \cdot \frac{\sin a}{R} \mp \sin A, \text{ when } y = \frac{h}{12} \cdot \frac{R^2}{\cos d}, \text{ which is called}$$

YASHṬI and can be found as follows.

$$y = \frac{h}{12} \cdot \frac{R^2}{\cos d} = \frac{R}{12} \cdot \frac{h}{12} \cdot \frac{12\,R}{\cos d},$$

$$= \frac{R}{12} \cdot \frac{h}{12} \left(12 + \frac{12 \text{ versed } d}{\cos d}\right).$$

When the BHUJA of the Sun's longitude is 15, 30, 45, 60, 75, 90, the value of $\frac{12 \text{ versed } d}{\cos d}$ is 4, 15, 32, 50, 63, 68 sixtieths respectively. The differences of these values are 4, 11, 17, 18, 13, 5 which are written in the text. Multiply these differences by h or the AKSHAKARṆA, divide the products by 12 and the quotients thus found are called the KHAṆḌAS for the given place. By assuming the BHUJA of the Sun's longitude as an argument, find the result through the KHAṆḌAS and take r for this result.

$$\text{Then } \frac{r}{60} = \frac{h}{12}\left(h + \frac{r}{60}\right),$$

$$\text{and hence, } \quad y = \frac{R}{12}\left(h + \frac{r}{60}\right).$$

But in this instrument $R = 30$

$$\therefore\ y = \frac{10}{4}\left(h + \frac{r}{60}\right) \text{which exactly coincide with the rule given in the}$$

text for determining the YASHṬI.

The value of the YASHṬI will certainly be more than 30, because the value of the AKSHAKARṆA or h is more than 12.

Now, (see the diagram) suppose m is the end of the YASHṬI in the PAṬṬIKÁ or index $o\,m$ which touches the circle in S, then, in the triangle $o\,m\,n$

$$R : o\,m = \sin m\,o\,n : m\,n;$$

$$\text{or } R : \quad y = \sin a \quad : m\,n;$$

$$\therefore m\,n = \frac{y \times \sin a}{R};$$

and hence, $\cos p = m\,n + \sin A,$

Q

LAMBA-REKHÁ, but below and above according as it was to be set off above or below in finding the time from the shadow, (this operation being the reverse of the former). The sine met by the sine of ascensional difference, thus set off, is the new sine across which the PAṬṬIKÁ or index is now to be placed till the YASHṬI-CHINHA or point of YASHṬI falls on it. This position will assuredly exhibit the place of the shadow of the axis.

The Yashṭi or Staff.

28, 29 and 30. Having drawn a circle (as the horizon) with a radius equal to radius of a great circle, mark east and west points (and the line joining these points is called the PRÁCHYAPARÁ or east and west line) and mark off (from them) the amplitude at the east and west. Draw a circle from the same centre with a radius equal to cosine of declination i. e. with a radius of diurnal circle, and mark this circle with 60 GHAṬIS. Now take the YASHṬI, equal to the radius (of the great circle) and hold it with its point to the Sun, so that no shadow be reflected from it; the other point should rest in the centre. Now measure the distance from the end of the amplitude to the point of the YASHṬI when thus held opposite to the Sun. This distance applied as a chord within the interior circle will cut off, if it be before midday, an arc of the number of GHAṬIKÁs from sun-rise, and if after midday an arc of the time to sun-set.*

that is, the sine of the ascensional difference is subtracted from or added to $m\,n$ the distance between the end of the YASHṬI and the middle sine, as the Sun be in the north or the south to the equinoctial.

Again, by taking $m\,r$ equal to sin A we have,

$$\begin{aligned} \cos p &= m\,n \mp \sin A = m\,n \mp m\,r, \\ &= n\,r \text{ or } t\,t', \\ &= \cos o\,t, \\ \therefore\ p &= o\,t \text{—B. D.]} \end{aligned}$$

* [It is plain from this, that the distance from the point of the staff to the end of the amplitude is the chord of the arc of the diurnal circle passing through the Sun, intercepted between the horizon and the Sun. For this reason, the arc subtended by the distance in question in this interior circle described with a radius of the diurnal circle which is equal to the cosine of the declination, will denote the time after sun-rise or to sun-set.—B. D.]

31. The perpendicular let fall from the point of the YASHṬI is the S'ANKU or sine of altitude: the place between the S'ANKU and centre is equivalent to DṚIGYÁ or sine of zenith distance. The sine of amplitude is the line between the point of horizon at which the Sun rises or sets, on which the point of the YASHṬI will rest at sun-rise and sun-set, and the east and west line the PRÁCHYAPARÁ.

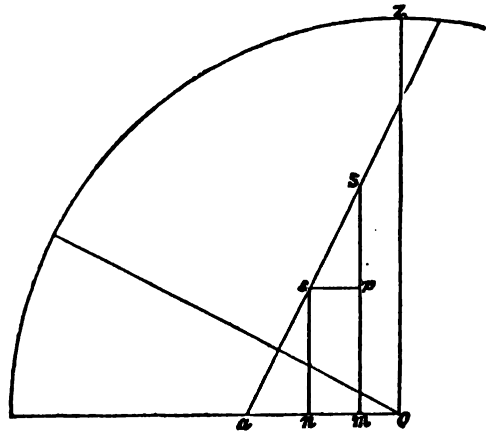
To find the PALABHA with the YASHṬI.

32 and 33. The distance between the S'ANKU and the UDAYÁSTA-SÚTRA, multiplied by 12 and divided by the S'ANKU, will be the PALABHÁ.

Take two altitudes of the Sun with the YASHṬI: observe the S'ANKUS of the two times and the BHUJAS.

Add the two BHUJAS, if one be north and the other south, or subtract if they be both of the same denomination: multiply the above quantity (whether sum or difference) by 12 and divide by the difference of the two S'ANKUS, the result will be the PALABHÁ.* The difference between the east and west line and the root of S'ANKU is called BHUJA.

* [Let O be the east or west point of the horizon O *a*, Z the zenith, *a s* S the



diurnal circle on which S and *s* are the Sun's two places at different times and S *m* and *s n* the S'ANKUS or the sines of altitudes of the Sun, then O *m*, O *n* will be the BHUJAS, *n m* or *s p* the difference between the BHUJAS and S *p* the difference between the S'ANKUS.

If the S'ANKU be observed three different times by the YASHṬI, then the time, declination &c. may be found (by simply observing the Sun).

To find PALABHA', declination, time, &c. from three observations by the YASHṬI, of three S'ANKUS.

34. First of all find three S'ANKUS: draw a line from the top of the first to the top of the last; from the top of the second S'ANKU, draw a line to the eastern point and a line to the western point of the horizon, so as to touch the first line drawn.

35. A line drawn so as to connect these two points in the horizontal circumference will be the UDAYA'STA SÚTRA. The distance between it and the centre will give the sine of amplitude. The line drawn through the centre parallel to the UDYA'STA-SÚTRA at the distance of the sine of amplitude is the east and west line.*

36. Find the PALABHÁ as before (and also the AKSHA-KARṆA). Now the sine of amplitude multiplied by 12 and divided by AKSHA-KARṆA will be the sine of declination. This again multiplied by the radius and divided by the sine of 24° or the sine of the Sun's greatest declination, will give the sine of the BHUJA of the Sun's longitude.

37 and 38. Which converted into degrees is Sun's longitude, if the observation shall have been made in the 1st quarter of the year. If in the second quarter, the longitude will be found by subtracting the degrees found from 6 signs: if

Now as the triangles $s\ q\ n$ and $S\ q\ m$ are the latitudinal triangles, the triangle $S\ s\ p$ is also the latitudinal

$\therefore\ Sp : sp = 12 :$ PALABHA'

$$\therefore \text{PALABHA'} = \frac{12\,sp}{Sp}.$$

It is when S, s two places of the Sun are both north or both south to the prime vertical, but when one place is north and other is south, the sum of the BHUJAS is taken.—B. D.]

* [As it is plain that the tops of the three S'ANKUS are in the plane of the diurnal circle, the line therefore drawn from the top of the first S'ANKU to that of the last, will also be in the same plane and hence the two lines touching this line, drawn from the top of the middle S'ANKU one to eastern and the other to western point of the horizon, lie in this plane. Therefore, the line joining these two points of the horizon is the intersecting line of the plane of the diurnal circle and that of the horizon, and consequently it is the UDAYA'STA SÚTRA.—B. D.]

in the 3rd quarter, 6 signs must be added: if in the fourth quarter of the year, then the degrees found must be subtracted from 12 signs for the longitude.

The quarters of the year will be known from the seasons, the peculiarities of each of which I shall subsequently describe.

It is declared (by some former astronomers) that the shadow of the gnomon revolves on the circle passing through the ends of the three shadows made by the same gnomon (placed in the centre of the horizon), but this is wrong, and consequently the east and west and north and south lines, the latitudes &c. found by the aid of the circle just mentioned are also wrong.*

39. Whether the place of the Sun be found from the shadow or from the sine of the amplitude, it will be found corrected for precession. If the amount of precession be subtracted, the Sun's true place will be found. If the true place of the Sun be subtracted, the amount of precession will be ascertained.

The praise of instrument called DHÍYANTRA or genius instrument.

40. But what does a man of genius want with instruments about which numerous works have treated? Let him only take a staff in his hand, and look at any object along it, casting his eye from its end to the top, there is nothing of which he will not then tell its altitude, dimensions, &c. if it be visible, whether in the heavens, on the ground or in the water on the earth.

Now I proceed to explain it.

41. He who can know merely with the staff in his hand, the height and distance of a bamboo, of which he has observed the root and top, knows the use of that instrument of instruments—genius—(the DHÍYANTRA) and tell me what is there that

* The existence of such gross error in the principles of a calculation as are here referred to as existing in the works of BHA'SKARA's predecessors would seem to indicate that the science of astronomy was not of more recent cultivation than Mr. Bentley and others have maintained.—L. W.

he cannot find out. [Here the ground is supposed to be perfectly level.]

42. Direct the staff lengthways to the north polar star; let drop-lines fall from both ends of staff, when thus directed to the star. Now the space between the two drops is the BHUJA or base of a right angled triangle, when the difference between the lines thus dropped is the KOṬI or perpendicular.

To find PALABHÁ.

43. The KOṬI multiplied by 12 and divided by the BHUJA gives the PALABHÁ.*

Having in the same way observed the root of the bamboo; [and in so doing found the BHUJA and KOṬI], multiply the BHUJA by the height of the man's eye.

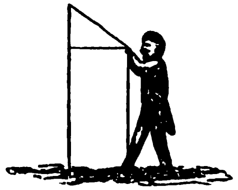
44 and 45. And divide the product by the KOṬI, the result is, you know the distance to the root of the bamboo.

To find the distance and height of a bamboo.

Having thus observed the top of the bamboo (with the staff, and ascertained the BHUJA and KOṬI), multiply the distance to the root of the bamboo by the KOṬI, and divide the product by the BHUJA, the result is the height of the bamboo above the observer's eye: this height added with the eye's height will give the height of the whole bamboo.†

For instance, suppose the staff 145 digits long, the height of observor's eye 68 digits; that in making the lower observation the BHUJA = 144 digits = 6 cubits, and KOṬI = 17 digits; that in making the observation of the top of the bamboo, the BHUJA =

Example.



* i. e. If this BHUJA : gives the KOṬI
: : 12 digits of gnomon : gives the PALABHA'.

† The observer first directs *a b* his staff to *d*, the root of the tree: The staff

116 digits and KOṬI = 87 digits. Then tell me the height of bamboo and the distance of it. As,

$$\frac{68 \times 144}{17} = 576 \text{ digits or 24 cubits distance to bamboo;}$$

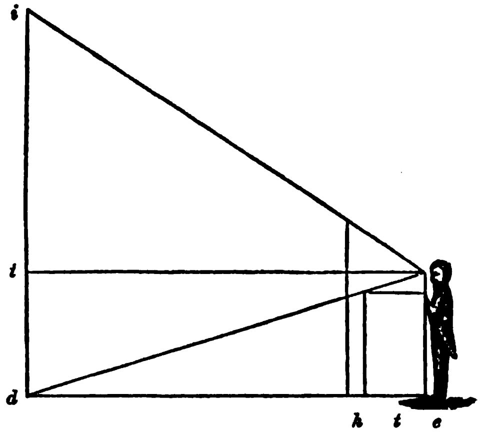
and $\frac{576 \times 87}{116} = 432$ height of tree above observer's eye,

68 add the eye's height,

500 height of tree.

Let a man, standing up, first of all observe the top of an object: then (with a staff, whether it be equal to the former or not in length), let him observe again the top of the same object whilst sitting.

46. Then divide the two KOṬIS by their respective BHUJAS: take the difference of these quotients, and by it divide the difference of the heights of observer's eye—this will give the distance to the bamboo: from this distance the height of the bamboo may be found as before.*



is furnished at either end with drop lines $a\,h$, $b\,k$: $b\,k - a\,h = b\,c =$ sin of $\angle\, b\,a\,c$. Then say

As $b\,c : a\,c :: b\,e : d\,e = f\,b$.

He then observes the top of object and finds $g\,f$, which is easy, as $f\,b$ has been found.—L. W.

* BHÁSKARA founds this rule on the following algebraic process.

Question.

47. There is a high famous bamboo, the lower part of which being concealed by houses &c. was invisible: the ground, however, was perfectly level: If you, my friend, remaining on this same spot by observing the top (first standing and then sitting), will tell me the distance and its height, I acknowledge you shall have the title of being the most skilful of observers and expert in the use of the best of instruments DHÍYANTRA.

Example.

The observer, first standing, observes the top of the bamboo and finds the BHUJA, with the first staff, to be 4 cubits or 96 digits: he then sits down and finds with another staff the BHUJA to be 90 digits. In both cases the KOṬI was one digit. Tell me, O you expert in observation, the distance of observer from the bamboo and the bamboo's height.

Observation in water.

48. So also the altitude may be observed in the surface of smooth water: but in this case the height of observer's eye is to be subtracted to find the true height of the object:—Or the staff may be altogether dispensed with: In which last case two heights of the observer's eye (viz. when he stands and sits) will be two KOṬIS: and the two distances from the observer to the

Let x = base, distance to bamboo. Then say

if $96 : 1 :: x : \frac{x}{96}$: then $\frac{x}{96} + 72$ = height of bamboo.

By second observation $90 : 1 :: x : \frac{x}{90}$, then $\frac{x}{90} + 24$ = height of bamboo.

Then $72 + \frac{x}{96} = 24 + \frac{x}{90}$; $\frac{x}{90} - \frac{x}{96} = 48$, or $\frac{6x}{8640} = 48$

$\therefore x = 69{,}120$ digits
$= 2880$ cubits.

That is $\frac{x}{90} - \frac{x}{96} = 72 - 24$

or $x = \frac{72 - 24}{\frac{1}{90} - \frac{1}{96}}$ that is difference of observer's height—difference of two KOṬIS divided by their respective BHUJA'S.—L. W.

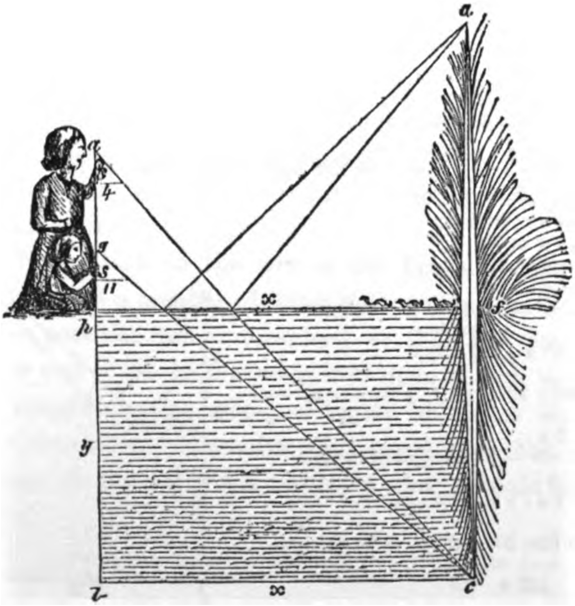
places in the water where the top of the object is reflected, the BHUJAS.

49. Having seen only the top of a bamboo reflected in water, whether the bamboo be near or at a distance, visible or invisible, if you, remaining on this same spot, will tell me the distance and height of bamboo, I will hold you, though appearing on Earth as a plain mortal, to have attributes of superhuman knowledge.

Question.

An observer standing up first observes (with his staff) the reflected top of a bamboo in water. The KOṬI $=3$ digits and BHUJA $=4$ digits. Then sitting down he makes a second observation and finds the BHUJA $=11$ digits and KOṬI $=8$ digits. His eye's height standing $=3$ cubits or 72 digits, and sitting $=1$ cubit or 24 digits. Tell me height of bamboo and its distance.*

Example.



* Let $df = fc =$ height of bamboo $= hb$
then ba or $y =$ height of bamboo and man's height together.
Let $bc =$ breadth of water $= x$
then by first observation

Example. A man standing up sees the shadow of a bamboo in the water—the point of the water at which the shadow appears is 96 digits off: then sitting down on the same spot he again observes the shadow and finds the distance in the water at which it appears to be 33 digits: tell me the height of the bamboo and his distance from the bamboo.*

$4 : 3 :: x : y$ or $3x = 4y$ or $x = \frac{4y}{3}$

by 2nd observation $11 : 8 :: x : y - 48$ digits

or $8x = 11y - 528$ or $x = \frac{11y - 528}{8}$

thus $x = \frac{4y}{3}$ and $x = \frac{11y - 528}{8}$

$\therefore \frac{4y}{3} = \frac{11y - 528}{8}$ or $4y = \frac{33y - 1584}{8}$

or $32y = 33y - 1584$, or $y = 1584$

$\therefore 1584 - 72 = 1512$ digits $= 63$ cubits $=$ height of bamboo.

2nd part. To find width of water or x

$$x = \frac{4y}{3} = \frac{1584 \times 4}{3} = 212 \text{ digits} = 88 \text{ cubits.—L. W.}$$

* Let $oe = 96$ digits
$cd = 33$
$ac = 72$
$bc = 24$

let $x =$ distance from observer to bamboo.

Now $ce : ac = jh : ja$

or $96 : 72 = x : y = \frac{72x}{96} = \frac{3x}{4}$

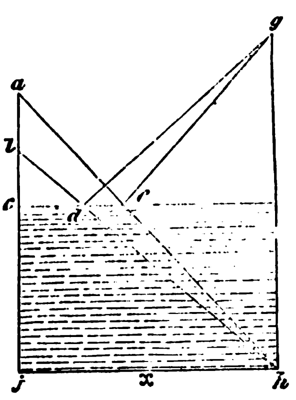
Then $\frac{3x}{4} - 3 =$ height of bamboo

Again $cd : bc :: jh : jb$

or $33 : 24 :: x : y - 48 = \frac{24x}{33}$

$= \frac{8x}{11}$

then $\frac{8x}{11} - 1 =$ height of bamboo



50 and 51. Make a wheel of light wood and in its circumference put hollow spokes all having bores of the same diameter, and let them be placed at equal distances from each other; and let them also be all placed at an angle somewhat verging from the perpendicular: then half fill these hollow spokes with mercury: the wheel thus filled will, when placed on an axis supported by two posts, revolve of itself.

A self-revolving instrument or SWAYANVAHA YANTRA.

Or scoop out a canal in the tire of the wheel and then plastering leaves of the TÁLA tree over this canal with wax, fill one half of this canal with water and other half with mercury, till the water begins to come out, and then cork up the orifice left open for filling the wheel. The wheel will then revolve of itself, drawn round by the water.

Make up a tube of copper or other metal, and bend it into the form of an ANKUS'A or elephant hook, fill it with water and stop up both ends.

Description of a syphon.

54. And then putting one end into a reservoir of water, let the other end remain suspended outside. Now uncork both ends. The water of the reservoir will be wholly sucked up and fall outside.

55. Now attach to the rim of the before described self-revolving wheel a number of water-pots, and place the wheel and these pots like the water-wheel so that the water from the lower end of the tube flowing into them on one side shall set the wheel in motion, impelled by the additional weight of the pots thus filled. The water discharged from the pots as they reach the bottom of the revolving wheel, should be drawn

$$\frac{8x}{11} - 1 = \frac{3x}{4} - 3 \text{ or } 2 = \frac{3x}{4} - \frac{8x}{11} = \frac{x}{44}$$

$$\therefore\ x = 44 \times 2 = 88$$

$$\text{Then } y = \frac{3x}{4} = \frac{3 \times 88}{4} = 3 \times 22 = 66, \text{ height of bamboo.}$$

off into the reservoir before alluded to by means of a water-course or pipe.

56. The self-revolving machine (mentioned by LALLA &c.) which has a tube with its lower end open is a vulgar machine on account of its being dependant, because that which manifests an ingenious and not a rustic contrivance is said to be a machine.

57. And moreover many self-revolving machines are to be met with, but their motion is procured by a trick. They are not connected with the subject under discussion. I have been induced to mention the construction of these, merely because they have been mentioned by former astronomers.

End of Chapter XI. called YANTRÁḌHYÁYA.

CHAPTER XII.

Description of the seasons.

Spring.

1. (This is the season in which) the KOKILAS (Indian black birds) amidst young climbing plants, thickly covered with gently swaying and brilliantly verdant sprouts of the mango (branches) raising their sweet but shrill voices say, "Oh travellers! how are you heart-whole (without your sweethearts, whilst all nature appears revelling) in the jubilee of spring CHAITRA, and the black bees wander intoxicated by the delicious fragrance of the blooming flowers of the sweet jasmine!"

2. The spring-born MALLIKÁ (Jasminum Zambac, swollen by the pride she feels in her own full blown beautiful flowers) derides (with disdain her poor) unadorned (sister) MÁLATÍ (Jasminum grandiflorum) which appears all black soiled and without leaf or flower (at this season), and appears to beckon her forlorn sister to leave the grove and garden with her

tender budding arms, agitated by the sweet breezes from the fragrant groves of the hill of MALAYA.

The GRÍSHMA or mid-summer season.

3. In the summer (which follows), the lovers of pleasure and their sweethearts quitting their stone built houses, betake themselves to the solitude of well wetted cottages of the KUS'AKÁS'A grass, salute each other with showers of rose-water and amuse themselves.

4. Now fatigued by their dalliance with the fair, they proceed to the grove, where KÁMA-DEVA has erected the (flowering) mango as his standard, to rest (themselves) from the glare of the fierce heat, and to disport themselves in the (well shaded) waters of its BOWRIS (or large wells with steps).

Rainy season.

5. (The rainy season has arrived, when the deserted fair one thus calls upon her absent lover :) Why, my cruel dear one, why do you not shed the light of your beaming eye upon your love-sick admirer? The fragrance of the blooming MÁLATÍ and the turbid state of every passing torrent proclaims the season of the rains and of all-powerful love to have arrived. Why, therefore, do you not have compassion on my miserable lot?*

6. (Alas, cries the deserted wife, alas!) the peacocks (delighted by the thundering clouds) scream aloud, and the breeze laden with the honied fragrance of the KADAMBA comes softly, still my sweet one comes not. Has he lost all delight for the sweet scented grove, has he lost his ears, has he no pity—has he no heart?

7. Such are the plaintive accusations of the wife in the season of the rains, when the jet black clouds overspread the sky:—angered by the prolonged absence of him who reigns over her heart, she charges him, but still smilingly and sweetly, with being cruelly heedless of her devoted love.

* This is one of those verses in which a double or triple meaning is attempted to be supported: to effect this, several letters however are to be read differently. —L. W.

8. The mountain burning with remorse at the guilt of having received the forbidden embraces of his own PUSHPAVATÍ daughter, forest appears in early autumn through its bubbling springs and streams sparkling at night with the rays of the Moon, to be shedding a flood of mournful tears of penitence.

The SÁRATKA'LA or season of early autumn.

9. In the HEMANTA season, cultivators seeing the earth smiling with the wide spread harvest, and the grassy fields all bedecked with the pearl-like dew, and teeming with joyous herds of plump kine, rejoice (at the grateful sight).

HEMANTA or early winter.

10. When the S'IS'IRA season sets in what unspeakable beauty and what sweet and endless variety of red and purple does not the 'KACHNÁR' grove unceasingly present, when its leaf is in full bloom, and its bright glories are all expanded.

S'IS'IRA or close of winter.

11. The rays of the Sun fall midday on the earth, hence in this S'IS'IRA season, they avail not utterly to drive away the cold:

* * * *

12. Here, under the pretence of writing a descriptive account of the six seasons, I have taken the opportunity of indulging my vein for poetry, endeavouring to write something calculated to please the fancy of men of literary taste.

Sweets of poetry.

13. Where is the man, whose heart is not captivated by the ever sweet notes of accomplished poets, whilst they discourse on every subject with refinement and taste? or whose heart is not enchanted by the blooming budding beauties of the handsome willing fair one, whilst she prattles sweetly on every passing topic:—or whose substance will she not secure by her deceptive discourse?

14. What man has not lost his heart by listening to the pure, correct, nightingale-like notes of the genuine poets? or who, whilst he listens to the soft notes of the water-swans on

the shores of large and overflowing lakes well filled with lotus flowers, is not thereby excited?

15. As holy pilgrims delight themselves, in the midst of the streams of the sacred Ganges, in applying the mud and the sparkling sands of its banks, and thus experience more than heaven's joys: so true poets lost in the flow of a fine poetic frenzy, sport themselves in well rounded periods abounding in displays of a playful taste.

End of Chapter XII.

CHAPTER XIII.

Containing useful questions called PRAS'NÁDHYÁYA.

Object of the Chapter and its praise.

1. Inasmuch as a mathematician generally fails to acquire distinction in an assemblage of learned men, unless well practised in answering questions, I shall therefore propose a few for the entertainment of men of ingenuity, who delight in solving all descriptions of problems. At the bare proposition of the questions, he, who fancies in his idle conceit, that he has attained the pinnacle of perfection, is often utterly disconcerted and appalled, and finds his smiling cheeks deserted of their colour.

2. These questions have been already put and have been duly answered and explained either by arithmetical or algebraic processes, by the pulverizer and the affected square, i. e. methods for the solutions of indeterminate problems of the first and of the second degree, or by means of the armillary sphere, or other astronomical instruments. To impress and make them still more familiar and easy I shall have to repeat a few.

3. All arithmetic is nothing but the rule of proportion: and Algebra is but another name for ingenuity of invention. To the clever and ingenious then what is not known! I, however, write for men and youths of slow comprehension.

Praise of ingenious persons.

4. With the exception of the involution and evolution of the square and cube roots, all branches of calculation may be wholly resolved into the rule of proportion. It indeed assumes many shapes, but it is universally prevalent. All this arithmetical calculation denominated PÁṬÍ GAṆITA, which has been composed in many ways by the wisest of former mathematicians, is only for the enlightenment of simple men like myself.

5. Algebra does not consist in the letters (assumed to represent the unknown quantities): neither are the different processes any part of its essential properties. But Algebra is wholly and simply a talent and facility of invention, because the faculties of inventive genius are infinite.

6. Why, O astronomer, in finding the AHARGAṆA, do you add SAURA months to the lunar months CHAITRA &c. (which may have elapsed from the commencement of the current year): and tell me also why the (fractional) remainders of ADHIMÁSAS and AVAMAS are rejected: for you know that to give a true result in using the rule of proportion, the remainders should be taken into account.

Question 1st.

7. If you have a perfect acquaintance with the MIS'RA or allegation calculations, then answer this question. Let the place of the Moon be multiplied by one, that of the Sun by 12 and that of Mars by 6, let the sum of these three products be subtracted from three times the Jupiter's place, then I ask what are the revolutions of the planet whose place when added to or subtracted from the remainder will give the place of Saturn?

Question 2nd.

8 and 9. In order to work this proposition in the first place proceed with the whole numbers of revolutions of the several planets in the KALPA, adding, subtracting and multiplying them in the manner mentioned in the question: then subtract the result from the revolutions of the planet given: or subtract the revolutions of the given planet from the result, according as the place of the unknown planet happen to be directed to be added or subtracted in the question. This remainder will represent the number of revolutions of the unknown planet in the KALPA. If the remainder is larger than the number from which it is to be subtracted, then add the number of terrestrial days in a KALPA, or if the remainder exceed the number of terrestrial days in the KALPA, then reduce it into the remainder by dividing it by the number of days in the KALPA.*

RULE.

* BHA'SKARA'CHA'RYA himself has given the following example in his commentary VA'SANA'-BHA'SHYA

Suppose Moon to have 4 revolutions in a KALPA of 60 days

Sun,........ 8 ....
Mars, ... 5 ....
Jupiter, ... 7 ....
Saturn, ... 9 ....

Then $4 \times 1 + 8 \times 12 + 5 \times 6 = 70$ and $7 \times 3 = 21$.
As 70 cannot be subtracted from 21 add 60 to it $= 81$,
Subtract 70,

remainder 11:

let $p =$ revolutions of the unknown planet, then by the question $11 - p = 9$
or $11 - 9 = 2 = p$,
but $11 + p = 9$ or $p = 9 - 11 = 60 + 9 - 11 = 58$:

It thus appears that the unknown planet has 2 or 58 revolutions in the KALPA.

Now let us see if this holds true on the 23rd day of this KALPA:

	revolutions		signs			
						s
for Moon, if	60 : 4	: :	23 : 6	.. 12°	this × 1 =	6 .. 12°,
Sun,	60 : 8	: :	23 : 1	.. 24	this × 12 =	9 .. 18,
Mars,	60 : 5	: :	23 : 11	.. 0	this × 6 =	6 .. 0
					signs	10 .. 0 subtracted
Saturn,	60 : 9	: :	23 : 5	.. 12		
Jupiter,	60 : 7	: :	23 : 8	.. 6	this × 3 =	0 .. 18 from
for p,	60 : 2	: :	23 : 9	.. 6	this sub. from	2 .. 18 remainder
						9 .. 6
					corresponding with Saturn,	5 .. 12

Question 3rd.

10. The algebraical learned, who knowing the sum of the additive months, subtractive days elapsed and their remainders, shall tell the number of days elapsed from the commencement of the KALPA, deserves to triumph over the student who is puffed up with a conceit of his knowledge of the exact pulverizer called SAM'SLISHṬA united, as the lion triumphs over the poor trembling deer he tears to pieces in play.

RULE.

11. For the solution of this question, you must multiply the given number of additive months, subtractive days and their remainders, by 863374491684 and divide by one less than the number of lunar days in a KALPA i. e. by 1602998999999, the remainder will be the number of lunar days elapsed from the beginning of the KALPA. From these lunar days the terrestrial days may be readily found.*

or if, 60 : 58 : : 23 : $\overset{s}{2}$: $\overset{\circ}{24}$. Then 2 .. 24 added
to 2 .. 18

still gives Saturn's place 5 .. 12

When $p = 9 - 11$, then as 11 cannot be subtracted from 9 the sum of 60 is added to the 9. The reason for adding 60 is that this number is always be denominator of the fractional remainder in finding the place of the planets; for the proposition.

If days of KALPA : revolutions : : given days give : here the days of KALPA are assumed to be 60 hence 60 is added.—L. W.

* [When the additive months and subtractive days and their remainders are given to find the AHARGAṆA.

Let $l = 1602999000000$ the number of lunar days in a KALPA.

$e = 159300000$ the number of additive months in a KALPA.

$d = 25082550000$ the number of subtractive days in a KALPA.

$A =$ additive months elapsed.

$A' =$ their remainder.

$B =$ subtractive days elapsed.

$B' =$ their remainder.

$a =$ the given sum of the elapsed additive months, subtractive days and their remainders.

and $x =$ lunar days elapsed;

then say $$\text{As } l : e : : x : A + \frac{A'}{l};$$

$$\text{As } l : d : : x : B + \frac{B'}{l};$$

12. Given the sum of the elapsed additive months, subtractive days and their remainders, equal (according to BRAHMAGUPTA'S system) to 648426000171; to find the AHARGAṆA. He who shall answer my question shall be dubbed a "BRAHMA-SIDDHÁNTA-VIT" i. e. shall be held to have a thorough knowledge of the BRAHMA-SIDDHÁNTA.*

Example.

$\therefore$ As $l : e + d :: x : A + B + \frac{A' + B'}{l}$ or $y + \frac{A' + B'}{l}$

$\therefore$ $(e + d)\,x = l\,y + A' + B'$, or $(e + d)\,x - l\,y = A' + B'$,
and $y = A + B$;

$\therefore$ by addition, $(e + d)\,x - (l - 1)\,y = A + B + A' + B'$,
$= a$;

by substitution, $26675850000\,x - 1602998999999\,y = a$:
now let, $26675850000\,x' - 1602998999999\,y' = 1$,
then we shall have by the process of indeterminate problems
$x' = 863374491684$.

Again, let $m = e + d$ and $n = l - 1$,
then $m\,x - n\,y = a$; (1)
and $m\,x' - n\,y' = 1$;
$\therefore$ $a\,m\,x' - a\,n\,y' = a$,
and $m\,n\,t - m\,n\,t = 0$:
$\therefore$ $m\,(a\,x' - n\,t) - n\,(a\,y' - m\,t) = a$:
which is similar to (1);
$\therefore$ $x = a\,x' - n\,t$
$= 863374491684\,a - (l - 1)\,t$.
Hence the rule in the text.—B. D.]

* Solution. The given sum $= 648426000171$ and the lunar days in a KALPA $= 1602999000000$:

$$\therefore \frac{648426000171 \times 863374491684}{1602998999999} = 349241932336$$ and 10300 remainder:

$\therefore$ 10300 these are lunar days elapsed.

To reduce them to their equivalent in terrestrial days says

If lunar days in a KALPA : Number of subtractive days :: Lunar days above found : 161 subtractive days and remainder amounting 267426000000.

$\therefore$ From	10300	Lunar days
subtract	161	Subtractive days
remainder	10139	Terrestrial days or AHARGAṆA.

Now to find additive months elapsed.

If lunar days in a KALPA : additive months of KALPA :: lunar days 10300 : 10 additive months and remn. 381000000000.

10 additive months $= 300$ lunar days.

$\therefore$ $10300 - 300 = 100{,}00$ SAURA days elapsed.

Hence 27 years 9 months and 10 days elapsed from the commencement of KALPA.—L. W.

13 and 14. Given the sum of the remainders of the revolutions, of the signs, degrees, minutes and seconds of the Moon, Sun, Mars, Jupiter, the S'ÍGHROCHCHAS of Mercury and Venus and of Saturn according to the DHÍVRIDDHIDA, including the remainder of subtractive days in finding the AHARGAṆA, abraded (reduced into remainder by division) by the number of terrestrial days (in a YUGA). He who, well-skilled in the management of SPHUṬA KUṬṬAKA (exact pulverizer), shall tell me the places of the planets and the AHARGAṆA from the abraded sum just mentioned, shall be held to be like the lion which longs to make its seat on the heads of those elephant astronomers, who are filled with pride by their own superior skill in breaking down and unravelling the thick mazes and wildernesses which occur in mathematical calculations.

Question 4th.

15. If the given sum abraded by the number of terrestrial days in a YUGA, on being divided by 4, leaves a remainder, then the question is not to be solved. It is then called a KHILA or an "impossible" question. If, on dividing by 4, no remainder remain, then multiply the quotient by 293627203, and divide the product by 394479375. The number remaining will give the AHARGAṆA. If the day of the week does not correspond with that of the question, then add this AHARGAṆA to the divisor (394479375) until the desired day of the week be found.*

RULE.

* [According to the DHÍVRIDDHIDA TANTRA of LALLA the terrestrial days in a YUGA = 1577917500 and the sum of all the 36 remainders for one day = 118407188600968 : this abraded by the terrestrial days in a YUGA = 259400968.

Let x = AHARGAṆA then say

As 1 : 259400968 : : x : 259400968 × x

This abraded by 1577917500 the terrestrial days in a YUGA will be equal to 1491227500 the given abraded sum of the 36 remainders, now

let y = the quotient got in abrading 259400968 x by 1577917500, then

$$259400968\,x - 1577917500\,y = 1491227500.$$

It is evident from this that as the coefficients of x and y are divisible by 4, the given remainder 1491227500 also must be divisible by 4, otherwise the question will be impossible as stated in the text.

Hence, dividing the both sides of the above question by 4,

$$64850242\,x - 394479375\,y = 372806875 : \ldots\ldots\ldots\ldots \quad (A)$$

and let $64850242\,x' - 394479375\,y' = 1, \ldots\ldots\ldots\ldots\ldots\ldots \quad (B)$

16. Tell me, my friend, what is the AHARGAṆA when on a Thursday, Monday or Tuesday, the 35 remainders of the revolutions, signs, degrees, minutes and seconds of the places of the planets, (the Sun, the Moon, Mars, Jupiter and Saturn and the s'ÍGHROCHCHAS of Mercury and Venus) together with the remainder of the subtractive days according to the DHÍVRIDDHIDA, give, when abraded by the number of terrestrial days in a YUGA, a remainder of 1491227500.*

Example.

17. The place of the Moon is of such an amount, that

Question 5th.

$$\frac{\text{The minutes}}{2} + 10 = \text{the seconds}$$

$$\text{the minutes} - \text{seconds} + 3 = \text{degrees}$$

$$\frac{\text{the degrees}}{2} = \text{signs.}$$

$\therefore$ $x' = 293627203$ by the processes of indeterminate problems.
Now let $a = 64850242$, $b = 394479375$, and $c = 372806875$;
$\therefore$ we have the equations (A) and (B) in the forms

$$a\,x - b\,y = c:$$
and $$a\,x' - b\,y' = 1,$$
$\therefore$ $x = c\,x' - b\,t$ (see the preceding note)
$= 293627203\ c - 394479375\ t$:
as stated in the text.—B. D.]

* Solution. The given sum of the 36 remainders in a YUGA = 1491227500 according to the DHÍVRIDDHIDA TANTRA.

$\therefore$ $1491227500 \div 4 = 372806875$:

and $\therefore$ $\dfrac{372808875 \times 293627203}{394479375} = 277495471$ and remainder 10000 i. e. AHARGAṆA.

$\therefore$ $\dfrac{10000}{7} = 1428 - 4$ remainder, i. e. 10000 = AHARGAṆA on a Tuesday, for the YUGA commenced on Friday.

This would be the AHARGAṆA on a Tuesday.

To find the AHARGAṆA on Monday, it would be necessary to add the reduced terrestrial days in a YUGA to this 10000, till the remainder when divided by 7 was 3.

$\dfrac{10000 + 394479375 \times 2}{7} = \dfrac{788968750}{7} = 112900821 - 3$ remainder = Monday:

and $\dfrac{10000 + 394479375 \times 3}{7} = \dfrac{1183448125}{7} = 169064017 - 6$ remainder or = Thursday.—L. W.

And the signs, degrees, minutes and seconds together equal to 130. On the supposition that the sum of these four quantities is of this amount on a Monday then tell me, if you are expert in rules of Arithmetic and Algebra, when it will be of the same amount on a Friday.*

18. Reduce the signs, degrees and minutes to seconds, adding the seconds, then reducing the terrestrial days and the planet's revolutions in a KALPA to their lowest terms, multiply the seconds of the planet (such as the Moon) by the terrestrial days (reduced) and divide by the number of seconds in 12 signs: then omitting the remainder, take the quotient and add 1 to it, the sum will be the remainder of the BHAGAṆAS revolutions.†

RULE.

* Let x = minutes

then $\frac{x+20}{2}$ = seconds

$x - \frac{x+20}{2} + 3$ = degrees.

$\frac{x - \frac{x+20}{2} + 3}{2}$ = signs

and $x + \frac{x+20}{2} + x - \frac{x+20}{2} + 3 + \frac{x - \frac{x+20}{2} + 3}{2} = 130$

$\therefore x = 58$ minutes.

$\frac{58+22}{2} = 39$ seconds.

$58 - 39 + 3 = 22$ degrees.

$\frac{22}{2} = 11$ signs.

Hence the Moon's place $= 11s. .. 22^{\circ} .. 58' .. 39''$.

† The mean place of the Moon $= 11s. .. 22^{\circ} .. 58' .. 39'' = 1270719''$
The number of seconds in 12 signs $= 1296000$.

Terrestrial days in a KALPA = 1577916450000 } These divided by 1650000 become DRIḌHA or reduced. { 956313
Revolutions of Moon = 57753300000 } { 35002.

19. The remainder before omitted subtracted from the divisor will give the remainder of seconds: if that remainder of the seconds is greater than the terrestrial days in a KALPA, then the question is an "impossible one" (incapable of solution and the planet's place cannot be found at any sunrise): but if less it may be solved. Then from the remainder of the seconds the AHARGAṆA may be found (by the KUṬṬAKA pulverizer as given in the LÍLÁVÁTÍ and BÍJA-GAṆITA) Or,

20. That number is the number of AHARGAṆA by which the reduced number of revolutions multiplied, diminished by the remainder of the revolutions and divided by the reduced number of terrestrial days in the KALPA, will bear no remainder. The reduced number of terrestrial days in a KALPA should be added to the AHARGAṆA such a number of times as may make the day of the week correspond with the day required by the question.

Now when the mean place of the Moon was sought, the rule was

As the Terrestrial days in a KALPA. : Revolutions in a KALPA. :: Given days or AHARGAṆA. : Revolutions.

If any remainder existed, it, when multiplied by the number of seconds in 12 signs and divided by KALPA, terrestrial days gave the Moon's mean place in seconds. We now wish to find the BHAGAṆA-S'ESHA or the remainder of revolutions, from the Moon's given place in seconds: we must therefore reverse the operation

$$\text{or } \frac{\text{Moon's place in seconds} \times \text{KALPA terrestrial days}}{\text{seconds in 12 signs}} = \text{BHAGAṆA-S'ESHA}.$$

The terrestrial days, however, to be used, must to be reduced to the lowest terms to which it, in conjunction with the KALPA-BHAGAṆAS or revolutions in a KALPA can be reduced: the lowest terms as above stated were of the terrestrial days = 956313, of the Moon's KALPA-BHAGAṆAS = 35002.

$$\therefore \frac{1270719 \times 956313}{1296000} = \frac{1215205099047}{1296000} = 937658 \text{ quotient} - \text{remainder}$$
331047.

937658 quotient
1 adding one

gives 937659 for the BHAGAṆA-S'ESHA.

The reason for adding one is, that we have got a remainder of 331047, which we never could have had, if the original remainder had been exactly 937658, it must have been 1 more. This is therefore added: but the remainder of seconds may now be found—for it will be 12963000 — 331047 = 964953.

This remainder 964953 being greater than the terrestrial days reduced to lowest terms, viz. 956313, the question does not admit of being solved.—L. W.

21. If the Moon's BHAGAṆA-S'ESHA or the remainder after finding the complete revolutions admits of being divided by 1650000, without leaving any remainder, the question may then be solved: the reduced BHAGAṆA-S'ESHA on being multiplied by 886834 and divided by 951363, then the remainder will give the AHARGAṆA. The divisor should be added to this remainder till the day of the week found corresponds with that of the question.*

Impossible question.

22. The mean place of the Moon will never be at any sun-rise, equal to 0 signs, 5 degrees, 36 minutes and 19 seconds.

Question 6th.

23. When will the square of the ADHIMÁSA-S'ESHA remainder of the additive months, multiplied by 10 and the product increased by one, be a square: or when will the square of the ADHIMÁSA-S'ESHA decreased by one and the remainder divided by 10 be a square? The man who shall tell me at what period of the KALPA this

* [To find the AHARGAṆA from the Moon's BHAGAṆA-S'ESHA.
Let R = BHAGAṆA-S'ESHA,
T = 1577916450000 terrestrial days in a KALPA,
M = 57753300000 the Moon's revolutions in a KALPA,
x = AHARGAṆA.

Then, as T : M : : x : revolutions $+ \dfrac{R}{T}$ or $y + \dfrac{R}{T}$:

$$\therefore \quad Mx - Ty = R:$$

In this equation as M and T are divisible by 1650000, R must be divisible by the same number, otherwise the question will be KHILA or "impossible," as stated in the text.

∴ Dividing both sides of the equation by the number 1650000, we have $35002\,x - 956313\,y = R'$ or $M'x - T'y = R'$:

Now let $M'x' - T'y' = 1$: or $35002\,x' - 956313\,y' = 1$: hence we have $x' = 886834$;
and $x = R'x' - T\,t$ (see the note on the verse 11th)
$= 886834\,R' - 956313\,t$. Hence the rule in the text.

And, as the reduced BHAGAṆAS'ESHA = 937659 (see the preceding note) hence $937659 \times 886834 = 831547881606$:

This divided by 956313 will give as quotient 869555 (i. e. t) leaving a remainder of 257151 which should be the AHARGAṆA, but as the BHAGAṆAS'ESHA i. e. 937659 does not admit of being divided by 1650000 (the numbers by which the terrestrial days were reduced) it ought to have been KHILA or insoluble question: but BHÁSKARÁCHÁRYA here still stated this number to be the true AHARGAṆA.—B. D.]

will take place—will be humbly saluted even by the wise, who generally speaking, gaze about in utter amazement and confusion at such questions, like the bee that wanders in the boundless expanse of heaven without place of rest.

Remark on the preceding question.

24. (In working questions of KUṬṬAKA pulverizer, the augment must be reduced by the same number by which the BHÁJYA dividend and HARA divisor are reduced to their lowest terms, and when the augment is not reducible by the same number as the BHÁJYA and HARA, the question is always insoluble.) But here, in working questions of KUṬṬAKA, those acquainted with the subject should know that the given augment is not to be reduced, i. e. it belongs to the reduced BHÁJYA and HARA, otherwise in some places the desired answer will not be obtained, or in others the question will be impossible.*

* [The questions in the 23rd verse are the questions of the VARGA-PRAKRITI or the affected square, i. e. questions of indeterminate problems of the second degree.

1st question. Let $a =$ the ADHIMÁSA-S'ESHA :

then by question $10\,x^2 + 1 = y^2$.

In such questions the coefficient of x is called PRAKRITI, the value of x KANISHTHA, that of the augment KSHEPA and that of y JYESHTHA.

Now assume $y = m\,x + 1$,

$$\text{then } 10\,x^2 + 1 = (m\,x + 1)^2,$$
$$= m^2\,x^2 + 2\,m\,x + 1,$$
$$\therefore \qquad x = \frac{2\,m}{10 - m^2}.$$

Hence the rule given by BHÁSKARÁCHARYA in his Algebra Ch. VI., verse VI., for finding the KANISHṬHA where the KSHEPA is 1, is "Multiply any assumed number by 2 and divide by the difference between the square of the number and the PRAKRITI, the quotient will be the KANISHṬHA where the KSHEPA is 1."

$$\text{Now assume } m = 3, \text{ then } x = \frac{2\,m}{10 - m^2} = \frac{2 \times 3}{10 - 9} = 6 :$$

$$\text{and} \quad \therefore \quad y = \sqrt{10\,x^2 + 1} = \sqrt{361} = 19 :$$

$\therefore$ ADHIMÁSA-S'ESHA $= 6$.

From two sets, whether identical or otherwise, of KANISHṬHA, JYESHṬHA and KSHEPA belonging to the same PRAKṚITI, all others can be derived such as follows.

Let $a =$ PRAKRITI, and

$\left.\begin{matrix} x_1, y_1, \& b_1 \\ x_2, y_2, \& b_2 \end{matrix}\right\}$ the two sets of KANISHṬHA, JYESHTHA and KSHEPA, then we have

$$a\,x_1^2 + b_1 = y_1^2 ;$$
$$a\,x_2^2 + b_2 = y_2^2 :$$
$$\therefore \qquad b_1 = y_1^2 - a\,x_1^2,$$
$$b_2 = y_2^2 - a\,x_2^2 :$$

25. Tell me, O you competent in the spheric, considering it frequently in your mind for awhile, what is the latitude of the city (A) which is situated at a distance of 90° from UJJAYINÍ, and bears

Question.

and $\therefore \quad b_1 \times b_2 = (y_1^2 - a x_1^2)(y_2^2 - a x_2^2),$

$$= y_1^2 y_2^2 - a x_1^2 y_2^2 - a x_2^2 y_1^2 + a x_1^2 x_2^2 ;$$

$$\therefore \quad a x_1^2 y_2^2 + a x_2^2 y_1^2 + b_1 b_2 = y_1^2 y_2^2 + a^2 x_1^2 x_2^2 :$$

adding $\pm 2 a x_1 x_2 y_1 y_2$ to both sides

$$a x_1^2 y_2^2 \pm 2 a x_1 x_2 y_1 y_2 + a x_2^2 y_1^2 + b_1 b_2 = y_1^2 y_2^2 \pm 2 a x_1 x_2 y_1 y_2 + a^2 x_1^2 x_2^2.$$

or $\quad a (x_1 y_2 \pm x_2 y_1)^2 + b_1 b_2 = (y_1 y_2 \pm a x_1 x_2)^2 :$

thus we get a new set of KANIṢṬHA, JYESHṬHA and KSHEPA :

i. e. new KANISHṬHA $= x_1 y_2 \pm x_2 y_1$;

new JYESHṬHA $= y_1 y_2 \pm a x_1 x_2$;

and new KSHEPA $= b_1 b_2$:

Hence the Rule called BHAVANÁ given by BHÁSKARÁCHARYA in his Algebra Ch VI. verses III. & IV.

Now in the present question

$$x_1 = 6, y_1 = 19 \text{ and } b_1 = 1 ,$$

and also $\quad x_2 = 6, y_2 = 19$ and $b_2 = 1$:

$\therefore$ now KANISHṬHA $= 6 \times 721 + 228 \times 19 = 4326 + 4332 = 8658$;

now JYESHṬHA $= 721 \times 19 + 10 \times 6 \times 228 = 13699 + 13680 = 17379$;

and now KSHEPA $= 1 \times 1 = 1$.

Thus $x = 8658$ &c., according to the Bhávaná assumed.

The second question is

$$\frac{x^2 - 1}{10} = y^2 ,$$

or $\quad x^2 = 10 y^2 + 1$.

Here then we have an equation similar to the former one, but x^2 is now be in the place of y^2 and y^2 in the place of x^2.

$\therefore \quad x$ will be $= 19$,

or $\quad = 721$ &c.

Now given ADHIMASA-SESHA as found by the first case $= 6$. The proportion by which this remainder was got, was

if KALPA SAURA days : KALPA-ADHIMASAS : : x or elapsed SAURA days

$$: y + \frac{6}{\text{KALPA SAURA days}} .$$

$\therefore$ KALPA-ADHIMASAS $\times x =$ KALPA SAURA days $\times y + 6$

or $\quad y = \dfrac{\text{KALPA-ADHIMASAS} \times x - 6}{\text{KALPA SAURA days}}$.

From this we get a new question : "What are the integer values of x and y in this equation?" which question is one of the questions of KUṬṬAKA and in which the coefficient of the unknown quantity in the numerator is called BHAJYA or dividend, the denominator HARA or divisor and the augment KSHEPA.

It is clear that in this equation, if the augment be not divisible by the same number as the dividend and divisor, the values of x and y will not be integers, and hence the question will be insoluble. But here in order that no question should be insoluble, the author has stated that the dividend and divisor should always be taken, reduced to their lowest terms, otherwise the question will be insoluble.

As in the present question, if the dividend KALPA-ADHIMASAS and the divisor KALPA SAURA days be taken not reduced to their lowest terms, i. e. not divided by due east from that city (UJJAYÍNÍ)? What is the latitude of the place (B) distant also 90° from the city (A) and bearing due west from it? What also is the latitude of a place (C) also 90° from (B) and bearing N. E. from (B): and of the place (D) which is situated at a distance of 90° from (C) and bears S. W. from (C)?*

the number 300000, the question will be an impossible one, because the augment 6 is not divisible by the same number. For this reason the dividend and divisor must be taken here reduced to their lowest terms.

Hence, dividend = reduced KALPA-ADHIMÁSAS $= \dfrac{1593300000}{300000} = 5311$; and

divisor = reduced KALPA SAURA days $= \dfrac{1555200000000}{300000} = 5184000$.

$\therefore$ By substitution, $y = \dfrac{5311\,x - 6}{5184000}$,

which gives $x = 826746$ the elapsed SAURA days or 2276 years 6 months and 6 days.—B. D.]

* Let $a =$ the azimuth degrees,
$d =$ the distance in degrees between the two cities,
$p =$ PALABHA' at the given city,
$k =$ AKSHA-KARṆA,
and $x =$ the latitude of the other city.

Then $\sin x = \left(\dfrac{\sin d \times \cos a}{\text{Rad}} \pm \dfrac{\cos d \times p}{12}\right) \times \dfrac{12}{k}$.

Now in the 1st question, $a = 90°$, $d = 90°$, $p = 5$ digits, the PALABHA' at UJJAYINÍ, and $k = \sqrt{12^2 + 5^2} = 13$:

$$\therefore\ \sin x = \left(\frac{3438 \times 0}{3438} \pm \frac{0 \times 5}{12}\right) \times \frac{12}{13};$$
$$= (0 \pm 0) \times \tfrac{12}{13} = 0:$$
$$\therefore\ x = 0 = \text{latitude of (A) or of YAMAKOṬI}.$$

(2). In the second question, $a = 90°$, $d = 90°$, $p = 0$ digits at YAMAKOṬI, and $\therefore k = 12$:

$$\therefore\ \sin x = \left(\frac{3438 \times 0}{3438} \pm \frac{0 \times 0}{12}\right) \times \frac{12}{12};$$
$$= (0 \pm 0)\ \tfrac{12}{12} = 0:$$
$$\therefore\ x = 0 \text{ Latitude of city (B) or LANKA}.$$

(3). In the 3rd question, $a = 45°$, $d = 90°$, $p = 0$ at LANKÁ and $k = 12$:

$$\therefore\ \sin x = \left(\frac{3438 \times 2431}{3438} + \frac{0 \times 0}{12}\right) \times \frac{12}{12};$$
$$= (2431 + 0) \times 1 = 2431:$$

26 and 27. Convert the distance of YOJANAS (between the two cities, one is given and the other is that of which the latitude is to be found,) into degrees (of a large circle), and then multiply the sine and cosine of these degrees by the cosine of the azimuth of the other city and PALABHÁ at the given city, and divide the products by radius and 12 respectively. Take then the difference between these two quotients, if the other city be south of east of the given city; and if it be north of that, the sum of the quotients is to be taken. But the reverse of this takes place, if the distance between the cities be more than a quarter of the earth's circumference. The difference or sum of the quotients multiplied by 12 and divided by AKSHAKARṆA will give the sine of the latitude sought.*

RULE.

$\therefore \quad x = 45^\circ$ Latitude of city (C).

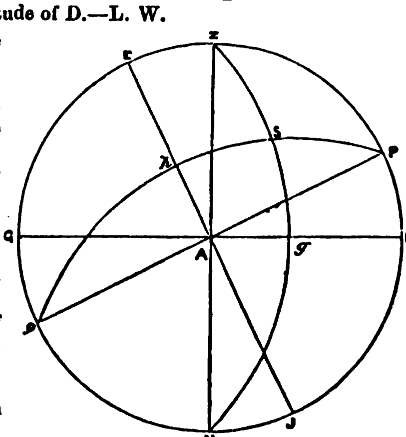
(4). In the 4th question, $a = 45^\circ$, $d = 90^\circ$, $p = 12$ at C and $\therefore k = 12\sqrt{2}$:

$$\therefore \quad \sin x = \left(\frac{3438 \times \frac{3438}{2}}{3438} \cdot \frac{\sqrt{2}}{} \sim \frac{0 \times 12}{12}\right) \times \frac{12}{12\sqrt{2}};$$

$$= \left(\frac{3438}{2}\sqrt{2} \sim 0\right) \times \frac{1}{\sqrt{2}} = \frac{3438}{2};$$

$\therefore \quad x = 30^\circ$ Latitude of D.—L. W.

* [Let Z be the Zenith of the given city bearing a north latitude, Z H N G the Meridian, G A H the Horizon, P the north pole, S the Zenith of the other city, the latitude of which is to be found and Z S N the azimuth circle passing through S. Then the arc Z S (which is equal to the distance in degrees between the two cities) will be the Zenith distance of S; the arc H G, the arc containing the given azimuth degrees, and S λ which is equal to the declination of the point S, the latitude of the other city which can be found as follows.



Let $a = $ H g the given azimuth degrees,

$d = $ Z S the distance in degrees between the two cities,

$p = $ PALABHA,

$k = $ AKSHA-KARṆA

28. Tell me quickly, O Astronomer, what is the latitude of a place (A) which is distant $\frac{1}{8}$ of the earth's circumference from the city of DHÁRÁ and bears 90° due east from it? What also is the latitude of a place distant 60° from DHÁRÁ, but bearing 45° N. E. from it? What also is the latitude of a place distant 60° from DHÁRÁ and bears S. E. from it? What also are the latitudes of three places 120° from DHÁRÁ and bearing respectively due east, N. E., and S. E. from it?*

Question.

and $x = S\,k$ the declination of the point S i. e. the latitude of the other city.
Then say, As sine Zg : sine $A\,g$: : sine Z S : the BHUJA i. e. the sine of distance from S to the Prime Vertical.

or $\quad R : \cos a : : \sin d :$ BHUJA

$$\therefore \text{BHUJA} = \frac{\cos a \sin d}{R}.$$

And by similar latitudinal triangles, $12 : p : : \cos d :$ S'ANKUTALA,

$$\therefore \text{S'ANKUTALA} = \frac{p \times \cos d}{12}.$$

Now when the other city is north of east of the given city, it is evident that the BHUJA will be north and consequently
the sine of amplitude = BHUJA + S'ANKUTALA :
but when the other city is south, the BHUJA also will be south and then, the sine of amplitude = BHUJA $\sim$ S'ANKUTALA,

$$\text{or the sine of amplitude} = \frac{\cos a \times \sin d}{R} \pm \frac{p \cos d}{12}.$$

And by latitudinal triangles
$k : 12 : :$ sine of amplitude : sine of declination i. e. $\sin x$

$$\therefore \sin x = \frac{12 \times \text{sine of amplitude}}{k} = \frac{12\left(\frac{\cos a \times \sin d}{R} \pm \frac{p \times \cos d}{12}\right)}{k},$$

hence the rule in the text.
If the distance in degrees between the two cities be more than 90°, the point S will then lie below the Horizon, and consequently the direction of the BHUJA will be changed. Therefore the reverse of the signs $\pm$ will take place in that case.—B D.]

* Here also $\sin x = \left(\frac{\sin d \times \cos a}{R} \pm \frac{\cos d \times p}{12}\right) \times \frac{12}{k}$.

(1.) In the first question, $a = 90°$, $d = 60°$, $p = 5$ digits the PALABHA of DHARA and $\therefore k = 13$.

$$\therefore \sin x = \left(\frac{2977 \times 0}{3438} + \frac{1719 \times 5}{12}\right) \times \frac{12}{13};$$

$$= \frac{1719 \times 5}{12} \times \frac{12}{13} = \frac{8595}{13} = 663\frac{9}{13}.:$$

29. Tell me, my friend, quickly, without being angry with me, if you have a thorough knowledge of the spheric, what will be the PALABHÁ of the city where the Sun being in the middle of the ARDRÁ NAKSHATRA (i. e. having the longitude 2 signs 13° 20′) rises in the north-east point.*

Question.

$\therefore\ x = 11^\circ..15'..1''$ Latitude of city due east from DHARA.

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(2). In the 2nd equation, $a = 45^\circ$, $d = 60^\circ$, $p = 5$ & $\therefore\ k = 13$:

$$\therefore\ \sin x = \left(\frac{2977 \times 2431}{3438} \times \frac{1719 \times 5}{12}\right) \times \frac{12}{13};$$

$$= \frac{19399109}{7449} = 2604\,\frac{1913}{7449};$$

$\therefore\ x = 49^\circ\ .18'..24''$ Latitude of city bearing 45° N. E. from DHARA.

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(3). In the 3rd question, $a = 45^\circ$, $d = 60^\circ$, $p = 5$ and $k = 13$.

$$\therefore\ \sin x = \left(\frac{2977 \times 2431}{3438} \sim \frac{1719 \times 5}{12}\right) \times \frac{12}{13};$$

$$= \frac{9549239}{7449} = 1281\,\frac{7070}{7449}.$$

$\therefore\ x = 21^\circ\ .54'..34''$ Latitude of city bearing the S. E. from DHARA.

---

(4). To find latitude of place 120° from DHARA and due east. Here, $\sin d = \sin 120^\circ = \sin 60^\circ = 2977$, $\cos d = \cos 120^\circ = -\sin 30^\circ = -1719$ $\cos a = 0$, $p = 5$ and k 13:

$$\therefore\ \sin x = \left(\frac{2977 \times 0}{3438} \pm \frac{1719 \times 5}{12}\right) \times \frac{12}{13};$$

$$= 662\,\frac{9}{13}:$$

$\therefore\ x$ or latitude $= 11^\circ\ .15'..1''$.

The latitudes of the places 120° bearing N. E. & S. E., will be the same as the latitudes of those places distant 60° and bearing S. E & N. E. Hence the latitudes are 21° 54′..34″ and 49° 18′ 24″.—L. W.

* *Ansr.* Sun's amplitude = sine of 45° = 2431′, the sine of longitude of middle of ARDRA = sine of 2 signs 13° 20′ = sin 73° 20′ = 3292′..6″ 40‴ and the sine of the Sun's greatest declination = sin 24° = 1397′.

Then say: As Rad : sin 24° : : sin (73° 20′) : sine of declination, and as sine of amplitude : sine of declination : : Rad : cos of latitude,

$\therefore$ sine of amplitude : sin 24° : : sin (73° 20′) : cos of latitude.

$$\therefore\ \text{cos of latitude} = \frac{\sin 24^\circ \times \sin (73^\circ..20')}{\text{sine of amplitude}} = \frac{1397' \times (3292'..6''\ .40''')}{2431'}$$

$$= 1891'\ 50'\ 48'' = \text{sine of } 33^\circ\ 23'\ 37'':$$

whence latitude will be 56° 36′ 23′ $\therefore$ sine of latitude = 2870′ 13″.

Then say: As cos of latitude : sine of latitude : : Gnomon : equinoctial shadow
1891′..51″ : 2870′ 13″ : : 12

$$\therefore\ \text{equinoctial shadow} = \frac{12 \times (2870'..13'')}{1891'\ 51''} = 18\,\frac{13}{60}\ \text{digits.—L. W.}$$

30. Tell me the several latitudes in which the Sun remains above the horizon for one, two, three, four, five and six months before he sets again.*

Question.

31. If you, O intelligent, are acquainted with the resolution of affected quadratic equations, then find the Sun's longitude, observing that the sum of the cosine of declination, the sine of declination, and the sine of the Sun's longitude: equal to 5000 is (the radius is assumed equal to 3438.)

Question.

32. Multiply the sum of the cosine of declination, the sine of declination, and the sine of Sun's longitude by 4, and divide the product by 15, the quotient found will be what has been denominated the ÁDYA. Next square the sum and double the square and divide by 337, the quotient is to be substracted from 910678. Take the square-root of the remainder. That root must then be subtracted from the ADYA above found: the remainder will be the declination, when the radius is equal to 3438. From the declination the Sun's longitude may be found.†

RULE.

* *Ansr.* When the Sun has northern declination he remains above the horizon for one month in 67° N. L.

two months in	69°
three months	73°
four months	78°
five months	84°
six months	90°

These are roughly wrought: for BHASKARÁCHÁRYA'S rule for finding these Latitudes see the TRIPRAS'NADHYAYAS of the GOLADHYAYA and also the GANITADHYAYA.—L. W.

† [Let $a =$ the given sum,
$p =$ the sine of the Sun's extreme declination
$x =$ the sine of the Sun's declination.

Then the cosine of declination will be $\sqrt{R^2-x^2}$ and the sine of the Sun's longitude $= \dfrac{R\,x}{p}$:

$\therefore$ by question $\sqrt{R^2-x^2} + x + \dfrac{R\,x}{p} = a$:

or $p\sqrt{R^2-x^2} + (R+p)\,x = a\,p$,

and $p\sqrt{R^2-x^2} = a\,p - (R+p)\,x$;

$\therefore R^2 p^2 - p^2 x^2 = a^2 p^2 - 2\,a\,p\,(R+p)\,x + (R^2 + 2\,R\,p + p^2)\,x^2$;

$\therefore (R^2 + 2\,R\,p + 2\,p^2)\,x^2 - 2\,a\,p\,(R+p)\,x = -(a^2 - R^2)\,p^2$;

33. Given the sum of the sines of the declination and of the altitude of the Sun when in the prime vertical; the TADDHRITI, the KUJYÁ and sine of amplitude equal to 9500, at a place where the PALABHÁ

Question.

or $$x^2 - \frac{2ap(R+p)}{R^2+2Rp+2p^2}x = -\frac{(a^2-R^2)p^2}{R^2+2Rp+2p^2};$$

completing the square, $$x^2 - \frac{2ap(R+p)}{R^2+2Rp+2p^2}x + \frac{a^2p^2(R+p)^2}{(R^2+2Rp+2p^2)^2}$$

$$= \frac{a^2p^2(R+p)^2}{(R^2+2Rp+2p^2)^2} - \frac{(a^2-R^2)p^2}{R^2+2Rp+2p^2};$$

$$= \frac{R^4p^2+2R^3p^3+2R^2p^4-a^2p^4}{(R^2+2Rp+2p^2)^2};$$

$$= \frac{R^2p^2}{R^2+2Rp+2p^2} - \frac{a^2p^4}{(R^2+2Rp+2p^2)^2}:$$

$$\therefore x - \frac{ap(R+p)}{R^2+2Rp+2p^2} = \pm\sqrt{\frac{R^2p^2}{R^2+2Rp+2p^2} - \frac{a^2p^4}{(R^2+2Rp+2p^2)^2}}:$$

or $$x = \frac{ap(R+p)}{R^2+2Rp+2p^2} \pm \sqrt{\frac{R^2p^2}{R^2+2Rp+2p^2} - \frac{a^2p^4}{(R^2+2Rp+2p^2)^2}}.$$

Now here $R = 3438$ and $p = 1397$,

$$\therefore \frac{ap(R+p)}{R^2+2Rp+2p^2} = \frac{ap(R+p)}{(R+p)^2+p^2} = \frac{a\times1397\times4835}{(4835)^2+(1397)^2} = \frac{6734495\,a}{25328834} =$$

$\frac{4}{15}a$ nearly = ÁDYA;

$$\frac{a^2p^4}{(R^2+2Rp+2p^2)^2} = \frac{3808777688881\,a^2}{641549831799556} = \frac{2a^2}{337} \text{ nearly};$$

and $\frac{R^2p^2}{R^2+2Rp+2p^2} = \frac{23067713928996}{25328834} = 910729$, in place of this the Author has taken the number 910678.

$\therefore x =$ ÁDYA $\pm\sqrt{910678 - \frac{2}{337}a^2}$:

but of these, the positive value is excluded by the nature of the case, because the sine of declination is always less than 1397.

Hence the Rule in the text.

Solution. The given sum = 5000,

$\therefore$ ÁDYA $= \frac{5000\times4}{15} = 1333'\ 20''$ and $\frac{2}{337}a^2 = 148367'\ 57''\ 9'''$.

$\therefore$ sine of declination $= 1333'\ 20'' - \sqrt{910678 - 148367'\ 57'\ 9''}$

$= 1333'\ 20'' - 873'\ 6'\ 13''$

$= 460'\ 13''\ 47'''$: from which we have the longitude of he Sun $= 0^s..19^\circ..14'\ 36''$ or $5^s\ 10^\circ..45'\ 24''$ or $6^s\ 19^\circ\ 14'..36''$ or $11^s..10^\circ..45'..24''$.—B. D.]

or equinoctial shadow is 5 digits, tell me then, my clever friend, if quick in working questions of latitudinal triangles and capable of abstracting your attention, what are the separate amounts of each quantity?

34. First assume the sine of declination to be equal to 12 times the shadow PALABHÁ: and then find the amounts of the remaining quantities upon this supposition. Then those on the supposition made, multiplied severally by the given sum and divided by their sum on the supposition made, will respectively make manifest the actual amounts of those quantities the sum of which is given.*

RULE.

35. If you have a knowledge of mathematical questions involving the doctrine of the sphere, tell me what will be the several amounts of sines of amplitude, declination and the KUJYÁ (where the PALABHÁ is 5 digits) when their sum is 2000.†

Question.

* Solution. Here PALABHÁ = 5 digits

∴ Suppose the sine of declination = 5 × 12 = 60:

and then say. If PALABHÁ : AKSHAKARṆA : : sine of decln. : SAMA S'ANKU

. or 5 : 13 : : 60 : SAMA S'ANKU $= \frac{13 \times 60}{5} = 156$,

Gnomon : AKSHAKARṆA : : SAMA S'ANKU : TADDHRITI $= \frac{156 \times 13}{12} = 169$,

12 : PALABHA' : : sine of decln. : KUJYA $= \frac{60 \times 5}{12} = 25$,

and 12 : AKSHAKARṆA : : sine of decln. : sine of amplitude $= \frac{60 \times 13}{12} = 65$.

∴ If the sum : sine of decln. supposed : : given sum : sine of decln. required.

or 475 : 60 : : 9500 : 1200.

If 475 : 156 : : 9500 : 3120 SAMA S'ANKU required.

and so on 3380 TADDHRITI

500 KUJYA

1300 sine of amplitude.

Ansr.

L. W.

† Solution. Here also PALABHA = 5,

then suppose sine of declination as before = 60,

and ∴ sine of amplitude = 65,

and KUJYA = 25,

the sum = 150,

36. But dropping for a moment those questions of the SIDDHÁNTAS involving a knowledge of the doctrine of the sphere, tell me, my learned friend, why in finding the point of the ecliptic rising above the horizon at any given time, (that is the LAGNA or horoscope of that time,) you first calculate the Sun's apparent or true place for that time, i. e. the Sun's instantaneous place: and further tell me, when the Sun's SÁVANA day, i. e. terrestrial day, consists of 60 sidereal GHAṬIKÁS and 10 PALAS, the LAGNA calculated for a whole terrestrial day should be in advance of the Sun's instantaneous place, and the LAGNA calculated for the time equal to the terrestrial day *minus* 10 PALAS should be equal to the Sun's instantaneous place.

Questions.

37. Are the GHAṬIKÁS used in finding the LAGNA, GHAṬIKÁS of sidereal or common SÁVANA time? If they are SÁVANA GHAṬIKÁS, then tell me why are the hours taken by the several signs of the ecliptic in rising, i. e. the RÁS'YUDAYA which are sidereal, subtracted from them, being of a different denomination? If on the other hand you say they are sidereal, then I ask why, in calculating the LAGNA for a period equal to a whole SÁVANA day i. e. 60 sidereal GHAṬIKAS and 10 PALAS, the LAGNA does not correspond with, but is somewhat in advance of, the Sun's instantaneous place; and then why the Sun's instantaneous place is used in finding the LAGNA or horoscope.*

38. Given the length of the shadow of gnomon at 10 GAṬÍS after sun-rise equal to 9 digits at a place where the PALABHÁ in 5 digits: tell me what is the longitude of the Sun, if you are *au fait* in solving questions involving a knowledge of the sphere.†

Question.

Then say as before

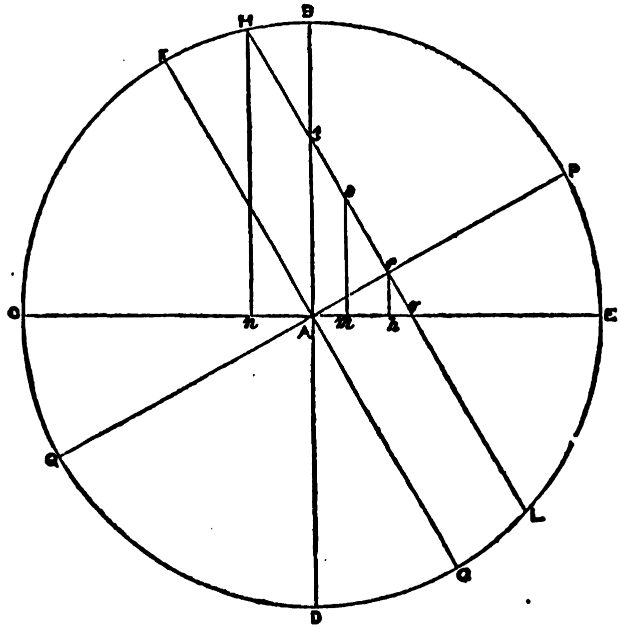
as 150 : 60 :: 2000 : 800 sine of declination,
as 150 : 65 :: 2000 : 866⅔ sine of amplitude,
as 150 : 25 :: 2000 : 333⅓ KUJYA.—L. W.

* [For answers to these questions see the note on the 27th verse of the 7th Ch.—B. D.]

† [For solving this question, it is necessary to define some lines drawn in the Armillary sphere and shew some of their relations.

39. Tell me, O Astronomer, what is the PALABHÁ at that place where the gnomon's shadow falling due west is equal to the gnomon's

Question.



Let B O D E be meridian of the given place, O A E the diameter of the Horizon, B the Zenith, P and Q the north and south poles, B A D the diameter of the Prime Vertical, F A G that of the Equinoctial, P A Q that of the six o'clock line, H f L that of one of the diurnal circles, s the Sun's projected place in it and $f\,h$, $s\,m$, H n perpendiculars to O E. Then

B F or E P = the latitude of the place,

A f = the sine of the Sun's declination,

A g = AGRA or the sine of amplitude,

$f\,g$ = KUJYA'. (It is called CHARAJYA' or sine of the ascensional difference when reduced to the radius of a great circle.)

$f\,s$ = KALA'. (It is called SÚTRA when reduced to the radius of a great circle.)

$s\,g$ = ISHṬA HṚITI. (It is called TADDHṚITI when s is at e, HṚITI when s is at H and KUJYA when s is at f.)

The ISHṬA HṚITI reduced to the radius of a great circle is called ISHṬA ANTYA', but s coincides with H, it is called ANTYA' only.

It is evident from the figure above described that

(1) ISHṬA HṚITI = KALA' ± KUJYA',

(2) ISHṬA ANTYA' = SÚTRA ± CHARAJYA',

(3) HṚITI = DYUJYA' or cosine of declination ± KUJYA',

(4) ANTYA' = radius ± CHARAJYA'.

Here the positive or negative sign is to be taken according as the Sun is in the northern or southern hemisphere.

height when the Sun is in the middle of the sign Leo, i. e. when his longitude is 4 signs and 15 degrees.*

Now at a given hour of the day, the ISHṬA HṚITI and others can be found as follows.

Half the length of the day diminished by the time from noon (or the NATA KA'LA properly so called) is the UNNATA KALA (or elevated time). Subtract from or add to the UNNATA KA'LA the ascensional difference according as the Sun is in the northern or southern hemisphere: reduce the remainder to degrees: the sine of the degrees is SÚTRA. The SÚTRA multiplied by the cosine of declination and divided by the radius gives the KALA'. Then from the above formulæ we can easily find the ISHṬA HṚITI and others.

Now to find the answer to the present question.

Square the length of the Gnomonic shadow and add it to the square of the Gnomon or 144: and square-root of the sum is called the hypothenuse of the shadow. From this hypothenuse find the MAHA'S'ANKU or the sine of the Sun's altitude by the following proportion.

As the hypothenuse of the shadow
: Gnomon or 12
: : Radius
: The MAHAS'ANKU or the sine of the Sun's altitude.

Then by similar latitudinal triangles,
as the Gnomon of 12 digits
: AKSHA KARṆA found from given PALABHA'
: : MAHAS'ANKU
: ISHṬA HRITI (see verses from 45 to 49 of the 7th Chapter).

Reduce the given UNNATA KA'LA to degrees and assume the sine of the degrees as ISHṬANTYA (for this will always be very near to the ISHṬA'NTYA). Then

$$\frac{\textit{cosine of declination}}{\text{Radius}} = \frac{\text{ISHTA HRITI}}{\text{ISHTA'NTYA}}$$

From this the cosine of declination will nearly be found, and thence the declination and ascensional difference can also be found. From the ascensional difference, just found, find the ISHTA'NTYA' of two kinds, one when the Sun is supposed to be in the northern hemisphere and the other when the Sun is supposed to be in the southern hemisphere. Of these two ISHṬANTYA'S that is nearly true which is nearer to the rough ISHṬA'NTYA' first assumed (i. e. the sine of the UNNATA KA'LA). From this new ISHṬA'NTYA' find again the declination and repeat the process until the roughness of declination vanishes. From the declination, last found, the longitude of the Sun can be found.—B. D.]

* The hypothenuse of the shadow is first to be found. Then say
As hypothenuse of the shadow
: Gnomon
: : Rad
: the MAHA' S'ANKU or the sine of the Sun's altitude.

Here we shall find sine of 45°. This is the SAMA S'ANKU.

It is 2431'

Sine of declination of the Sun when in 4 signs .. 15° = 987' 48"

$\therefore \overline{2431'}\,)^2 - \overline{987' .. 48''}\,)^2 = (\text{TADDHRITI} - \text{KUJYA'})^2$

or 5909761 — 975749 .. 9 = 4934011 51.

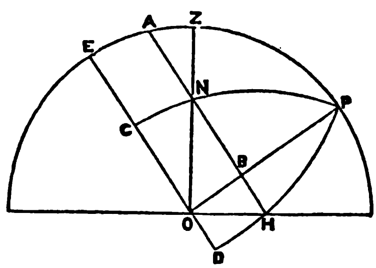
$\therefore \text{TADDHRITI} - \text{KUJYA'} = \sqrt{4934011 .. 51} = 2221' .. 15''$

Here we have 3 sides of the latitudinal triangle consisting SAMA S'ANKU, declination and TADDHRITI — KUJYA'. Hence we may find the latitude.

Then by similar latitudinal triangles
As TADDHRITI — KUJYA' 2221' .. 15'
: sine of declination 987' .. 48"
: : Gnomon 12
: PALABHA' 5½ digits.—L. W.

40. When the Sun enters the primo vertical of a person at UJJAYINÍ either at 5 GHATIS after sun-rise or 5 GHATIS before or after midday, what are his declinations? If you will answer me this I will hold you to be the sharp ANKUS'A (goad) for the guidance of the intoxicated elephants, the proud astronomers.*

Question.



* First of all assume H N the TADDHRITI = sine of the given elevated time that is = sin 30°. From this find the S'ANKU or the sine of altitude by similar triangles.

If AKSHA KARṆA or hypothenuse of equinoctial shadow.

: Gnomon 12

: : TADDHRITI

: SAMA S'ANKU $= \dfrac{12 \times \text{TADDHRITI}}{13} =$ O N

From O N, to find O B the sine of declination say

as AKSHA KARṆA : PALABHA' : : O N : O B $= \dfrac{\text{PALABHA}' \times \text{O N}}{13} =$ sine of declination.

From O B we may now find the longitude of the Sun and O D the ascensional difference: Now deduct this ascensional difference from the sine of elevated time converted into degrees. Hence

$$C D - O D = C O.$$

Now reduce C O to terms of a small circle on the supposition that the Sun has the declination now found.

As Rad : C O : : cosine of declination : N B.

Now find also B A by the same proportion.

Then N B + B A = N H' a new value of TADDHRITI.

If H N : gave O B : : H N' : O B' corrected value of O B.

Hence a corrected longitude of the Sun.

The operation to be repeated till rightness is found.

2nd.—To find the declination from the NATA KA'LA or time from noon = sin 30°.

Let a = the sine of NATA KA'LA : $R^2 - a^2 =$ SÚTRA^2,

and x = the sine of declination : $R^2 - x^2 = \cos^2$ of declination.

The SÚTRA reduced to value of diurnal circle will give KALA'

The proportion is. As R : SÚTRA : : cos of declination : KALA',

but I do not know what cos of declination is but only its square.

I must therefore make this proportion in squares

As R^2 : SÚTRA^2 : : cos^2 of declination : $\overline{\text{KALA}})^2 = \dfrac{(R^2 - a^2)(R^2 - x^2)}{R^2}$

Now by similar latitudinal triangles

As $\overline{12})^2 : \overline{\text{PALABHA}'})^2 : : \overline{\text{KALA}})^2$: sine^2 of declination

$\therefore$ sine^2 of declination $= \dfrac{\overline{\text{PALABHA}})^2}{\overline{12})^2} \times \overline{\text{KALA}})^2 = \dfrac{25}{144} \times \dfrac{(R^2 - a^2)(R^2 - x^2)}{R^2}$

$= x^2$

41. In a place of which the latitude is unknown and on a day which is unknown, the Sun was observed, on entering the prime vertical, to give a shadow of 16 digits from a gnomon (12 digits long) at 8 GHAṬIKÁs after sun-rise. If you will tell me the declination of the Sun, and the PALABHÁ I will hold you to be expert without an equal in the great expanse of the questions on directions space and time.*

Question.

42. O Astronomer, tell me, if you have a thorough knowledge of the latitudinal figures, the PALABHÁ and the longitude of the Sun

Question.

Now $R^2 - a^2 = 8864883$

$$25\,(R^2 - a^2) = 25 \times 8864883 = 221622075$$

and $144\ R^2 = 144\ (3438)^2 = 1702057536$

$$\therefore \frac{221622075\ (R^2 - x^2)}{1702057536} = x^2$$

$$R^2 - x^2 = \frac{1702057536}{221622075}\,x^2 = 7\tfrac{2}{3}\,x^2 \text{ nearly}$$

$$\therefore 26\,x^2 = 3\,R^2 : x^2 = \frac{3\,R^2}{26} = 1363828$$

and $x = \sqrt{1363828} = 1167' =$ sine of $19° .. 51'$

Hence the Sun's place may be found.—L. W.

* To find the sine of altitude or MAHA' S'ANKU

$(16)^2 + (12)^2 = (20)^2$ ... hypothenuse of the shadow $= 20$.

Then say

As $20 : 12 :: 3438' : 2062' .. 48'' =$ the MAHA' S'ANKU.

Now suppose the sine of UNNATA KA'LA or 8 GHAṬIKA's to be the TADDHṚITI $= 2655$.

Then by similar triangles

$$2062' .. 48'' : 2655' :: 12 : \text{AKSHA KARṆA} = \frac{2655' \times 12}{2062\tfrac{4}{5}}$$

From this find the PALABHA'.

To find declination says

As AKSHA KARṆA : PALABHA' :: $2062' .. 48''$: sine of declination.

From this find the cosine of declination, the KUJYA, the ascensional difference, &c. The UNNATA KALA diminished by the ascensional difference gives the time from 6 o'clock : the sine of this time will be the SÚTRA and hence the KALA : thence (KUJYA' being added) the TADDHṚITI : and thence the AKSHA KARṆA and declination. The operation to be repeated till the error of the original assumption vanishes.—L. W.

at that place, where (at a certain time) the KUJYÁ is equal to 245 and the TADDHṚITI is equal to 3125.*

43. Given the sum of the 3 following quantities, viz. of the sines of declination, and of the altitude of the Sun (when in the prime vertical) and of the TADDHṚITI decreased by the amount of the KUJYÁ equal to 6720, and given also the sum of the KUJYÁ, the sines of amplitude and declination (at the same time) equal to 1960. I will hold him, who can tell me the longitude of the Sun and also PALABHÁ from the given sums, to be a bright instructor of astronomers, enlightening them as the Sun makes the buds of the lotus to expand by his genial heat.†

Question.

* Ansr. Let x = the PALABHA

then say. As $x : 12 :: 245 :$ sine of declination $= \frac{2940}{x}$.

Now find the TADDHṚITI *minus* KUJYA'.

As $x : 12 :: \frac{2940}{x} :$ TADDHRITI — KUJYA $= \frac{35280}{x^2}$.

But TADDHRITI — KUJYA $= 3125 - 245 = 2880$.

$\therefore 2880 = \frac{35280}{x^2}$ and $x^2 = \frac{35280}{2880} = \frac{49}{4}$

$\therefore x = \frac{7}{2} = 3\frac{1}{2}$ PALABHA.

To find declination say

As $3\frac{1}{2} : 12 :: 245 : 840$ sine of declination.

Hence the longitude of the Sun may be discovered as before.—L. W.

† This question admits of a ready solution in consequence of its peculiarities.

$$\left.\begin{array}{r}\text{The sine of declination}\\ \text{SAMA S'ANKU}\\ \text{and TADDHṚITI — KUJYA}\end{array}\right\} = 6720$$

are all three respectively perpendiculars in the three latitudinal triangles.

$$\left.\begin{array}{r}\text{And the KUJYA}\\ \text{the sine of amplitude}\\ \text{and the TADDHṚITI — KUJYA}\end{array}\right\} = 1960$$

are bases in the same 3 triangles.

Hence we may take the sum of the 3 perpendiculars and also the sum of the three bases and use them to find the PALABHA.

As the sum of the 3 perpendiculars	sum of the 3 bases in the same triangles	Gnomon	PALABHA
6720 :	1960	:: 12 :	$\frac{1960 \times 12}{6720} = 3\frac{1}{2}$.

Now the KUJYA, sine of amplitude and sine of declination are the three sides of a latitudinal triangle. These three I may compare with the three Gnomon, PALABHA and AKSHA KARṆA to find the value of any one.

44. Given the sum of the sine of declination, sine of the Sun's altitude in prime vertical and the TADDHṚITI *minus* KUJYÁ equal to 1440′, and given also the sum of the sine of amplitude, the sine of the Sun's altitude in prime vertical and the TADDHṚITI equal to 1800′. I will hold him, who having observed the given sums.*

Question.

45. Given the equinoctial shadow equal to 9. What longitude must the Sun have in that latitude to give an ascensional difference of three GHAṬIS? I will hold you to be the best of astronomers if you will answer me this question.†

Question.

46. Hitherto it has been usual to find the length of the Sun's midday shadow, of the shadow of the Sun when in the prime verti-

Question.

But the AKSHA KARṆA must be first found to complete the sum of those three.

$$\text{AKSHA KARṆA} = \sqrt{(12)^2 + (\tfrac{7}{2})^2} = \sqrt{\frac{625}{4}} = \frac{25}{2}$$

$$\left.\begin{array}{l}\text{Gnomon} = 12\\ \text{PALABHA} = 3\tfrac{1}{2}\\ \text{AKSHA KARṆA} = 12\tfrac{1}{2}\end{array}\right\} = 28 \text{ sum of the 3 sides of a latitudinal triangle.}$$

Now if 28 : 12 : : 1960 : 840 the sine of declination.

Hence the place of the Sun as before.—L. W.

* This question is similar to the preceding.

In the first sum we have the sum of three perpendiculars in three different latitudinal Triangles. In the second we have the sum of the three hypothenuses of those same three Triangles. Hence we may say.

	sum 3 per.	sum of 3 corresponding hy.		Gnomon	AKSHAKARṆA
As	1440 :	1800	: :	12 :	15

Now from AKSHA KARṆA to find PALABHÁ

$$\text{PALABHÁ} = \sqrt{(15)^2 - (12)^2} = \sqrt{81} = 9.$$

Now sine of amplitude, sine of the Sun's altitude in the Primo Vertical, and the TADDHṚITI are the three sides of a latitudinal.—L. W.

† Let x = sine of the Sun's declination.

then $12 : 9 : : x : \text{KUJYÁ} = \frac{3}{4}x$.

Again $\sqrt{R^2 - x^2}$ = cosine of declination.

Then as R : cos of declination : : sine of ascensional differce. : KUJYÁ

Sine of ascensl. diffce. or CHARAJYÁ = sine of 3 GHAṬIS = sin $18° = 1062'$.

$$\therefore \frac{\text{cosin of decln.} \times \text{CHARAJYÁ}}{R} = \text{KUJYÁ}$$

$$\text{or } \frac{\sqrt{R^2 - x^2} \times 1062}{R} = \frac{3}{4}x$$

Hence may be found the sine of the Sun's decln. and thence his longitude.—L. W.

cal, and when in an intermediate circle (i. e. when he has an azimuth of 45°) by three different modes of calculation: now he who will by a single calculation tell me the length of those three shadows and of the shadows at any intermediate points at the wish of the querist, shall be held to be a very Sun on the Earth to expand the lotus-intellects of learned astronomers.*

* [Here the problem is this:—Given the Sun's declination or amplitude, the Equinoctial shadow of the place and the Sun's azimuth, to find the Sun's shadow.

For solving this problem BHÁSKARÁCHÁRYA has stated two different Rules in the GAṆITÁDHYÁYA. Of them, we now shew here the second.

"Multiply the square of the Radius by the square of the equinoctial shadow, and the square of the cosine of the azimuth by 144. The sum of the products divided by the difference between the squares of the cosine of the azimuth and the sine of the amplitude, is called the PRATHAMA (first) and the continued product of the Radius, equinoctial shadow and the sine of the amplitude divided by the (same) difference is called the ANYA (second). Take the square-root of the square of the ANYA added to the PRATHAMA: this root decreased or increased by the ANYA according as the Sun is in the northern or southern hemisphere gives the hypothenuse of the shadow (of the Sun) when the Sun is in any given direction of the compass."

"But when the cosine of the azimuth is less than the sine of the amplitude, take the square-root of the square of the ANYA diminished by the PRATHAMA: the ANYA decreased and increased (separately) by the square-root (just found) gives the two values of the hypothenuse (of the Sun's shadow) when the Sun is in the northern hemisphere."

This rule is proved algebraically thus.

Let $a =$ the sine of amplitude,

$A =$ the sine of azimuth,

$e =$ the Equinoctial shadow,

and $x =$ the hypothenuse of the shadow when the Sun is in any given direction of the compass.

Then say

as $x : 12 :: R :$ the MAHÁ S'ANKU or the sine of the Sun's altitude $= \frac{12\,R}{x}$

and $\therefore$ the sine of the Sun's zenith distance $= \sqrt{R^2 - \left(\frac{12\,R}{x}\right)^2} = \frac{R}{x}\sqrt{x^2-144}.$

Now, as $12 : e = \frac{12\,R}{x} :$ S'ANKUTALA $= \frac{e\,R}{x}$.

$\therefore$ BÁHU or the sine of an arc of a circle of position contained between the Sun and the Prime Vertical $= a \mp \frac{e\,R}{x}$: (see Ch. VII. V. 41) here the sign — or + is used according as the Sun is in the northern or southern hemisphere.

Then say

as $\frac{R}{x}\sqrt{x^2-144} : a \mp \frac{e\,R}{x} :: R : A :$

$\therefore \frac{R\,A}{x}\sqrt{x^2-144} = \left(a \mp \frac{e\,R}{x}\right) R;$

X

47. He who, knowing both the azimuth and the longitude of the Sun, observes one shadow of the gnomon at any time, or he who knowing the azimuth observes two shadows and can find the PALABHÁ, I shall conceive him to be a very GARUḌA in destroying conceited snakes of astronomers.

Question.

[On this BHÁSKARÁCHÁRYA has given an example in the GAṆITÁDHYÁYA as follows.

" Given the hypothenuse of the shadow (at any hour of the day) equal to 30 digits and the south BHUJA* equal to 3 digits: given also

Example.

or $A\sqrt{x^2-144} = ax \mp eR$;

$A^2x^2 - 144A^2 = a^2x^2 \mp 2Reax + e^2R^2$;

$(A^2-a^2)x^2 \pm 2Reax = e^2R^2 + 144A^2$;

$$x^2 \pm 2\frac{Rea}{A^2-a^2}x = \frac{e^2R^2+144A^2}{A^2-a^2};$$

or $x^2 \pm 2\,\text{ANYA}\,x = \text{PRATHAMA}$ (1)

$\therefore x^2 \pm 2\,\text{ANYA}\,x + \text{ANYA}^2 = \text{PRATHAMA} + \text{ANYA}^2$

and $\therefore x = \sqrt{\text{PRATHAMA} + \text{ANYA}^2} \mp \text{ANYA}$.

But when $A \angle a$ and the Sun is in the northern hemisphere, the equation (1) will be $x^2 - 2\,\text{ANYA}\,x = -\,\text{PRATHAMA}$,

and then $x = \text{ANYA} \pm \sqrt{\text{ANYA}^2 - \text{first}}$:

i. e. the value of the hypothenuse of the shadow will be of two kinds here.

Hence the Rule.

BHASKARACHARYÁ was the first Hindu who has given a general rule for finding the Sun's shadow *whatever* be the azimuth; and he was the first who has shewn that in certain cases the solution gives two different results.—B. D.]

* [On a levelled plane draw east and west and south and north lines and on their intersecting point, place Gnomon of 12 digits: the distance between the end of the shadow of that Gnomon and the east and west line is called the BHUJA.

It is to be known here that the value of the great BHUJA (as stated in 41st verse of the 7th Ch.) being reduced to the hypothenuse of the shadow becomes equal to the BHUJA (above found).

Or as the Radius

: the great BHUJA

: : the hypothenuse of the shadow

: the reduced BHUJA or the distance of the end of the shadow from the east and west line.

This reduced BHUJA is called north or south according as the end of the shadow falls north or south of the east and west line.

It is very clear from this that the reduced BHUJA will be the cosine of the azimuth in a small circle described by the radius equal to the shadow.

Or as the shadow

: the reduced BHUJA

: : radius of a great circle

: the cosine of the azimuth.

This is the method by which all Hindus roughly determine the azimuth of the Sun from the BHUJA of his gnomonic shadow.—B. D.]

the hypothenuse equal to 15 digits, and the north BHUJA equal to 1 digit, to find the PALABHÁ. Or, given the declination equal to 846 and only one hypothenuse and its corresponding BHUJA at the time, to find the PALABHÁ."]

RULE.

48. First of all multiply one BHUJA of the shadow by the hypothenuse of the other, and the second BHUJA by the hypothenuse of the first: then take the difference of these two BHUJAS thus multiplied, if they are both north or if both south, and their sum if of different denominations, and divide the difference or the sum by the difference of the two hypothenuses; it will be the PALABHÁ.*

Question.

49. How should he who, like a man just drawn up from the bottom of a well, is utterly ignorant of the PALABHÁ, the place of the Sun, the points of the compass, the number of the years elapsed from

* The rule mentioned here for finding the PALABHA' when the two shadows and their respective BHUJAS are given, is proved thus,

Let h_1 = the first hypothenuse of the shadow,

b_1 = its corresponding BHUJA,

h_2 = the second hypothenuse,

and b_2 = its corresponding BHUJA,

Then

As $h_1 : 12 :: R : \frac{12\,R}{h_1}$ = the first MAHA S'ANKU;

and in the same manner $\frac{12\,R}{h_2}$ = the second MAHA S'ANKU;

and also as $h_1 : b_1 :: R : \frac{b_1\,R}{h_1}$ = the first great BHUJA,

and $\therefore$ $\frac{b_2\,R}{h_2}$ = the second great BHUJA,

Then the PALABHA' $= \dfrac{\frac{b_1\,R}{h_1} \mp \frac{b_2\,R}{h_2}}{\frac{12\,R}{h_2} - \frac{12\,R}{h_1}}$ (see Ch. XI. V. 32)

$= \dfrac{b_1 h_2 \mp b_2 h_1}{h_1 - h}$

Hence the Rule.—L. W.

x 2

the commencement of the YUGA, the month, the TITHI or lunar day and the day of week, being asked by others to tell quickly the points of the compass, the place of the Sun, &c., give a correct answer? He, however, who can do so, has my humble reverence, and what astronomers will not acknowledge him worthy of admiration?*

Question.

50. He, who can know merely with the staff in his hand, the height and distance of a bamboo of which he has observed the root and top, knows the use of that instrument of instruments—Genius (the DHÍYANTRA): and tell me what is there that he cannot find out!

Question.

51. There is a high famous bamboo, the lower part of which, being concealed by houses, &c. was invisible: the ground, however, was perfectly level. If you, my friend, remaining on this same spot, by observing the top, will tell me the distance and its height, I acknowledge you shall have the title of being the most skilful of observers, and expert in the use of the best of instruments, DHÍYANTRA.

Question.

52. Having seen only the top of a bamboo reflected in water, whether the bamboo be near or at a distance, visible or invisible, if you, remaining on this same spot, will tell me the distance and height of the bamboo, I will hold you, though appearing on the Earth as a plain mortal, to have attributes of superhuman knowledge.†

53. Given the places of the Sun and the Moon increased by the amount of the precession of the equinox, i. e. their longitudes, equal to four and two signs (respectively) and the place of the Moon decreased by the place of the ascending node equal to 8 signs, tell me whether the Sun and the Moon have the same declination (either both south or one north

* This refers to the 34th verse of the Ch. XI.—L. W.
† [Answers to these questions will be found in the 11th Ch. —B. D.]

and one south), if you have a perfect acquaintance with the DHÍVRIDDHIDA TANTRA.

54. If the place of the Moon with the amount of the precession of the equinox be equal to 100 degrees, and the place of the Sun increased by the same amount to 80 degrees, and the place of the Moon diminished by that of the ascending node equal to 200 degrees, tell me whether the Sun and the Moon have the same declination, if you have a perfect acquaintance with the DHÍVRIDDHIDA TANTRA.

55. If you understand the subject of the PÁTA i. e. the equality of the declinations (of the Sun and the Moon), tell me the reason why there is in reality an impossibility of the PÁTA when there is its possibility (in the opinion of LALLA), and why there is a possibility when there is an impossibility of it (according to the same author).

56. If the places of the Sun and the Moon with the amount of the precession of the equinox be equal to 3 signs *plus* and *minus* 1 degree (i. e. 2*s*. 29° and 3*s*. 1° respectively) and the place of the Moon decreased by that of the ascending node equal to 11*s*. 28°, tell me whether the Sun and the Moon have the same declination, if you perfectly know the subject.

57. (In the DHÍVRIDDHIDA TANTRA), it is stated that the PÁTA *is to come* in some places when it has already taken place (in reality), and also it *has happened* where it *is to come*. It is a strange thing in this work when the possibility and impossibility of the PÁTA are also reversely mentioned. Tell me, O you best of astronomers, all this after considering it well.*

Date of the Author's birth and his work.

58. I (BHÁSKARA), born in the year of 1036 of the S'ÁLIVÁHANA era, have composed this SIDDHÁNTA-S'IROMAṆI, when I was 36 years old.

Author's apology.

59. He who has a penetrating genius like the sharp point of a large DARBHA straw, is qualified to compose a good work in mathe-

* [Answers to these questions will be found in the last Chapter of the GANITADAYAYA.—B. D.]

matics: excuse, therefore, my impudence, O learned astronomers, (in composing this work for which I am not qualified).

60. I, having lifted my folded hands to my forehead, beg the old and young astronomers (who live at this time) to excuse me for having refuted the (erroneous) rules prescribed by my predecessors; because, those who fix their belief in the rules of the predecessors will not know what is the truth, unless I refute the rules when I am going to state astronomical truths.

Author's birth-place, &c.

61. The learned MAHES'WARA, the head of all astronomers, the most good humoured man, the store of all sciences, skilful in the discussion of acts connected with law and religion, and a BRÁHMAṆA descended from S'ÁNḌILYA (a MUNI), flourished in a city, thickly inhabited by learned and dull persons, virtuous men of all sorts, and men competent in the three VEDAS, and situated near the mountain SAHYA.

62. His son, the poet and intelligent BHÁSKARA, made this clear composition of the SIDDHÁNTA by the favour of the lotus-like feet of his father; this SIDDHÁNTA is the guidance for ignorant persons, propagator of delight to the learned astronomers, full of easy and elegant style and good proofs, easily comprehensible by the learned, and remover of mistaken ideas.

63. I have repeated here some questions, which I have stated before, for persons who wish to study only this PRAS'NÁDHYÁYA.

64. The genius of the person who studies these questions becomes unentangled, and flourishes like a creeping plant watered at its root by the consideration of the questions and answers, by getting hundreds of leaves of clear proofs, shooting from the Spheric as from a bulbous root.

End of the 13th and last Chapter of the GOLÁDHYÁYA of the SIDDHÁNTA-S'IROMAṆI.

APPENDIX.

ON THE CONSTRUCTION OF THE CANON OF SINES.

1. As the Astronomer can acquire the rank of an A'CHARYA in the science only by a thorough knowledge of the mode of constructing the canon of sines, BHA'SKARA therefore now proceeds to treat upon this (interesting and manifold) subject in the hope of giving pleasure to accomplished astronomers.

2 and 3. Draw a circle with a radius equal to any number of digits: mark on it the four points of the compass and 360°. Now by dividing 90° by the number of sines (you wish to draw in a quadrant), you will get the arc of the first sine. This arc, when multiplied by 2, 3 &c., will successively be the arcs of other sines. Now set off the first arc on the circumference on both sides of one of the points of the compass and join the extremities of these arcs by a transverse straight line, the half of which should be known the sine of the first arc: All the other sines are thus to be known.

4. Or, now, I proceed to state those very sines by mathematical precision with exactness. The square-root of the difference between the squares of the radius and the sine is cosine.

5. Deduct the sine of an arc from the radius the remainder will be the versed sine of the complement of that arc, and the cosine of an arc deducted from the radius will give the versed sine of that arc. The versed sine has been compared to the

arrow between the bow and the bow-string: but here it has received the name of versed-sine.

6. The half of the radius is the sine of 30°: the cosine of 30° will then be the sine of 60°. The square-root of half square of radius will be the sine of 45°.

7. Deduct the square-root of five times the fourth power of radius from five times the square of radius and divide the remainder by 8: the square-root of the quotient will be the sine of 36°.

$$\text{Or } \sqrt{\frac{\text{rad}^2 \times 5 - \sqrt{\text{rad}^4 \times 5}}{8}} = \text{sine } 36°.*$$

8. Or the radius multiplied by 5878 and divided by 10000 will give the sine of 36°, (where the radius = 3438.) The cosine of this is the sine of 54°.†

9. Deduct the radius from the square-root of the product of

* [This is proved thus.

Let a = sine 18°; and ∴ R — a = covers 18° or vers 72°.

Then $\sqrt{\frac{R \times \text{vers } 72°}{2}} = \text{sine } \frac{72}{2}°$: (see the 10th verse.)

or $\sqrt{\frac{R\,(R - a)}{2}} = \text{sine } 36°$;

but $a = \frac{\sqrt{5\,R^2} - R}{4}$ (see the 9th verse)

$$\therefore \text{sine } 36° = \sqrt{\frac{R\left\{R - \frac{1}{4}\left(\sqrt{5\,R^2} - R\right)\right\}}{2}} = \sqrt{\frac{5\,R^2 - \sqrt{5\,R^4}}{8}}.\text{B. D.]}$$

† The Rule in 8th verse viz., $\frac{R \times 5878}{10000}$ seems to be the same as above and to be deduced from it;

for $\sqrt{\frac{5\,R^2 - \sqrt{5\,R^4}}{8}} = R.\sqrt{\frac{5 - \sqrt{5}}{8}}$

$\sqrt{5} = 2.237411$ &c.

and ∴ $5 - \sqrt{5} = 2.762589$ which divided by 8 = .345323

$$\therefore \text{sine } 36° = R\,\sqrt{.345323} = R \times .5878 = \frac{R \times 5878}{10000}.\text{—L. W.}$$

the square of radius and five and divide the remainder by 4: the quotient thus found will give the exact sine of 18°.*

10. Half the root of the sum of the squares of the sine and versed sine of any arc, is the sine of half that arc. Or, the sine of half that arc is the square-root of half the product of the radius and the versed sine.

11. From the sine of any arc thus found, the sine of half the arc may be found (and so on with the half of this last). In like manner from the complement of any arc may be ascertained the sine of half the complement (and from that again the sine of half of the last arc).

Thus the former Astronomers prescribed a mode for determining the other sines (from a given one), but I proceed now to give a mode different from that stated by them.

12. Deduct and add the product of radius and sine of BHUJA from and to the square of radius and extract the square-roots of the halves of the results (thus found), these roots will respectively give the sines of the half of 90° decreased and increased by the BHUJA.

In like manner, the sines of half of 90° decreased and increased by the KOṬI can be found from assuming the cosine for the sine of BHUJA.

13. Take the sines of BHUJAS of two arcs and find their difference, then find also the difference of their cosines, square

* [This is proved thus.

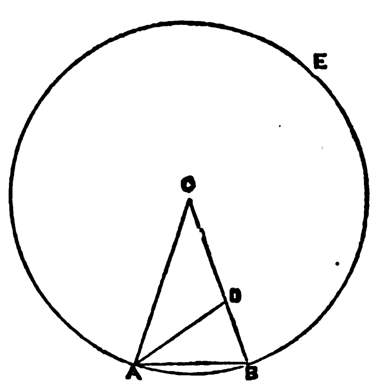
Let O be centre of the circle ABE and $\angle$ O = 36°, then AB = 2 sin 18°, and $\angle$s (OAB, OBA) each of them = 2 O.

Draw AD bisecting the $\angle$ OAB, then AB, AD, OD will be equal to each other.

Now let $x = \sin 18°$, then by similar triangles OB : AB = AB : BD or $R : 2x = 2x : R - 2x$;

$\therefore 4x^2 = R^2 - 2Rx$ which gives

$$x = \frac{\sqrt{5R^2} - R}{4}.\text{—B. D.]}$$



these differences, add these squares, extract their square-root and halve it. This half will be the sine of half the difference of the sines.* Thus sines can be determined by several ways.

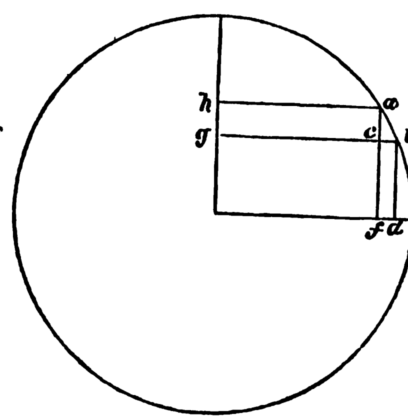
14. The square-root of half the square of the difference of the sine and the cosine of the BHUJA of an arc is equal to the sine of half the difference of the BHUJA and its complement.†

I will now give some rules for constructing sines without having recourse to the extraction of roots.

15. Divide the square of the sine of the BHUJA by the half radius. The difference between the quotient thus found and the radius is equal to the sine of the difference between the

* This rule is obvious, for ac = diffce. of sines bd & af and cb = diffce. of cosines bg & ah and $\overline{ac}\,)^2 + \overline{bc}\,)^2 = \overline{ab}\,)^2$
ab = chord of difference of arcs
$\frac{ab}{2}$ = sine of half that difference.

L. W.



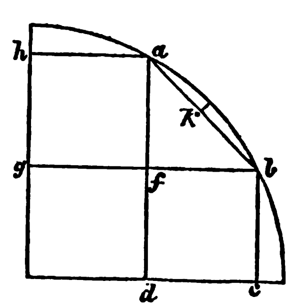
† Let bc = sine of any arc and bg = its cosine.

Draw the sine ad = cosine bg, then ah its sine will be equal to bc and $af = fb$:

$\therefore af^2 + fb^2 = ab^2$: but as $af^2 = fb^2$

$\therefore af^2 = \frac{ab^2}{2}$ and $\frac{af^2}{2} = \frac{ab^2}{4}$;

$\therefore \sqrt{\frac{af^2}{2}} = \frac{ab}{2}$.—L. W.]



degrees of BHUJA and its complement.* In this way several sines may be found here.

[As these several rules suffice for finding only the sines of arcs differing by 3 degrees from each other and not the sines of the intermediate arcs, the author therefore now proceeds to detail the mode of finding the intermediate sines, that is the sine of every degree of the quadrant. This mode, therefore, is called PRATIBHÁGAJYAKÁ-VIDHI.]

Rules for finding the sine of every degree from 1° to 90°.

16. Deduct from the sine of BHUJA its $\frac{1}{6569}$ part and divide the ten-fold sine of KOṬI by 573.

17. The sum of these two results will give the following sine (i. e., the sine of BHUJA one degree more than original BHUJA and the difference between the same results will give the preceding sine, i. e., the sine of BHUJA one degree less than original BHUJA). Here the first sine, i. e., the sine of 1°, will be 60 and the sines of the remaining arcs may be successively found.

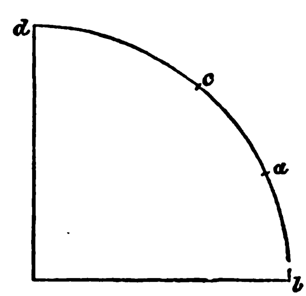
18. The rule, however, supposes that the radius = 3438. Thus the sines of 90° of the quadrant may be found.

Multiply the cosine by 100 and divide the product by 1529.

Rules for finding the 24 sines viz., of 3°¾, 7°½, 11°¼, 15°, &c.

19. And subtract the $\frac{1}{467}$ part of the sine from it. The sum of these will be the following sine (i. e., the sine of arc of 3°¾ degrees more than original arc): and the differ-

* Let ab be any arc, and $ac = ab$, then $ad =$ its complement, $cd =$ their difference, and $bc = 2\,ab$.



$$\text{Now } \sqrt{\frac{R \times \text{vers } bc}{2}} = \sin \frac{bc}{2} \text{ or } \sin ab,$$

$$\text{or } \frac{R \times \text{vers } bc}{2} = \sin^2 ab,$$

$$\text{or vers } bc = \frac{2 \sin^2 ab}{R},$$

$$\text{then } R - \text{vers } bc \text{ or } \sin cd = R - \frac{\sin^2 ab}{\frac{1}{2} R}. \text{—L. W.}$$

ence of them will be preceding sine (i. e., the sine of arc $3^{\circ}\frac{3}{4}$ degrees less than original arc).

20. But the first sine (or the sine of $3_{0}\frac{3}{4}$) is here equal to $224\frac{6}{7}$ (and not to 225 as it is usually stated to be). By this rule 24 sines may be successively found.*

21 and 22. If the sines of any two arcs of a quadrant be multiplied by their cosines reciprocally (that is the sine of the first arc by the cosine of the 2d and the sine of the 2d by the cosine of the first arc) and the two products divided by radius, then the quotients will, when added together, be the sine of the sum of the two arcs, and the difference of these quotients will be the sine of their difference.† This excellent rule called JYA-BHÁVANÁ has been prescribed for ascertaining the other sines.

Rules for finding the sines of sum and difference of any two arcs.

23. This rule is of two sorts, the first of which is called SAMÁSA-BHÁVANÁ (i. e., the rule for finding the sine of sum of two arcs) and the second ANTARA-BHÁVANÁ (i. e., the rule to find the sine of difference of arcs).

[If it be desired to reduce the sines to the value of any other radius than that above given of 3438.] Find the first sine by the aid of the above-mentioned rule PRATIBHÁGAJYAKÁ-VIDHI.

24 and 25. And then reduce it to the value of any new radius by applying the proportion. After that apply the JYÁ-BHÁVANÁ rule through the aid of the first sine and the cosine thus found, for as many sines as are required. The sines will thus be successively eliminated to the value of any new radius.

The rule given in my PATÍ or LÍLÁVATÍ is not sufficiently accurate (for nice calculations) I have not therefore repeated here that rough rule.

* [These rules given in the verses from 16 to 20 are easily deduced from the rules given in the verses 21 and 22.—B. D.]

† BHÁSKARÁCHÁRYA has given these rules in his work without any demonstration.—B. D.]

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